UNCERTAINTY AND MULTIPRODUCT MONOPOLY: SPILLOVERS ACROSS UNRELATED MARKETS

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ABSTRACT

When a monopolist produces unrelated products with known demand, the price of one good is invariant to changes in the cost of the other goods. However, if the demand for one good is uncertain, it is possible that the monopolist will subsidize increases in the cost of one good by charging a higher price for the other good. Surprisingly, under general assumptions, a monopolist will increase the price of a good with uncertain demand given higher production cost for a good with known demand.

INTRODUCTION

Multiproduct firms with market power allegedly cross subsidize. Airlines are accused of using revenue from their more profitable long-haul flights to cover the losses incurred in their short-haul routes with lower demand. Railroads supposedly use monies from their more profitable freight business to cover the losses in passenger service. Boeing allegedly subsidizes the lower prices charged on their smaller planes sold in more competitive markets with the monopoly profits they receive from selling their larger airframes. When Boeing merged with McDonnell-Douglas, Airbus was concerned with the possibility that Boeing would use its new-found revenue from military sales to cross-subsidize lower prices on their commercial planes.

Hospitals cross subsidize lesser priced, high cost, out-patient emergency room service with higher prices charged for hospital stays. Likewise, hospitals cross subsidize the care of indigents by charging higher prices to the insured and cash payers. Higher education subsidizes the cost of college by transferring monies from high-income, cash payers to financial aid for low-income students. Colleges also use the revenues from teaching to subsidize the cost of conducting research. Regulated monopolies misallocate costs incurred in their competitive enterprises to those of their regulated activities. The regulated businesses with their misallocated cost are subject to rate-of-return regulation, allowing the regulated firm to charge lower prices for the products sold in competitive markets.

This paper investigates whether a multiproduct monopolist subsidizes the increased cost of one product by charging a higher price for an unrelated product. Given two unrelated products where both demand curves are known with certainty, the price of one product is invariant to changes in the cost of the other product. However, once uncertainty is introduced, a monopolist that maximizes expected utility will increase the price of the good facing uncertain demand given an increase in the cost of an unrelated good which faces a certain demand curve. On the other hand, if the cost of producing the good with uncertain demand increases, the price of the good with certain demand is not affected.

Following these introductory comments, a brief literature review provides the foundation on which the model in this paper is based. The theoretical model is presented in the paper's third section. Conclusions and thoughts about future research are discussed in the fourth and final section of the paper.

REVIEW OF THE LITERATURE

This paper's genesis has roots in two branches of economic literature: the issues concerning price theory and uncertainty, and the output-price decisions of a multiproduct monopolist. These different areas of the literature are discussed in that order.

Price theory and uncertainty

Mills [9] analyzed the effect of uncertainty on a monopolist's pricing decisions, finding the results depend on the shape of the monopolist's marginal cost curve. For example, in the case of constant marginal cost, Mills determined that the monopolist's optimal price will be lower with uncertainty than without. In a dynamic theory that emphasizes the role of inventories of the finished product, Zabel [12] also examined the pricing behavior of a monopoly with uncertain demand. He demonstrates that the monopolist's optimal level of inventory will fall as the holding cost of inventory increases. Given a uniform distribution of demand and constant marginal cost, Zabel finds a monopoly will increase its price as the holding cost of inventory also increases.

In often-cited works, Sandmo [11, 12] and Berhardt [2] investigate the output decisions of a competitive firm under price uncertainty. They find two key facts. First, under price uncertainty, the output of the competitive firm is less than the output that would occur with price certainty. Secondly, if decreasing absolute Arrow-Pratt [1, 10] risk aversion is assumed, then a competitive firm with price uncertainty will reduce output as its fixed cost increases. Leland [6] extends Sandmo's result to the theory of monopoly under uncertainty. He finds that under uncertainty a monopolist's price and output decision are not invariant to changes in fixed cost. If the demand curve exhibits what Leland calls the "principle of increasing uncertainty," then the quantity-setting monopolist will produce a smaller output than the certainty amount that occurs where a known marginal revenue curve intersects the marginal cost curve.

Harris and Raviv [5] use a model of demand uncertainty, and they find that endogenously derived pricing schemes for a monopolist depend on capacity constraints. In their results, an optimal single price exists only if capacity constraints are not binding. Using a capital asset pricing model that explicitly includes risk, Brick and Jagpal [3] examine a monopoly's decisions regarding price and advertising under uncertainty. Not surprising, they find increases in demand leads to increases in the monopoly price. The optimal level of advertising, however, depends on how responsive the risk-adjusted price elasticity of demand is to changes in advertising.

Multiproduct firms

One of the first papers to study the implications of uncertain demand on a multiproduct monopoly was the work done by Dhrymes [4]. He decomposed this problem into two components. First, the monopolist determines the optimal output mix by maximizing expected utility. Next, given this optimal output mix, the monopoly determines the optimal combination of inputs by minimizing cost. Dhrymes concludes that the qualitative results of his model are similar to those of a uniproduct firm; but, the multiproduct firm's response to changes in the state of the uncertainty is more complex than that of a uniproduct firm. Exogenous shocks to the state of uncertainty include both changes in the firm's attitude toward risk and changes in the underlying probability distribution function that characterizes the firm's risk.

Meyer [7, 8] extends the analysis of a monopoly under uncertainty to a monopoly with multiple outputs and multiple inputs. In his 1975 paper dealing with simultaneous pricing and capacity decisions under uncertainty, Meyer [7] found that the optimal investment decision usually entailed some excess capacity. His 1976 paper applied components of the capital asset pricing theory to directly incorporate the market price of risk. In this paper, Meyer [8] found that the optimal pricing structure depended on the marginal risk associated with each distinct group of customers. One interesting result was his finding that optimal pricing may involve selling output to several groups of customers at a price below the marginal production cost.

THE MODEL

Suppose a profit-maximizing, quantity-setting monopolist sells two, unrelated products, goods X and Y, in separate markets. The monopolist knows the demand for good Y with certainty; however, the demand for good X is uncertain. Symbolically, x denotes the amount of good X sold, y is the output of good Y, p_X is the market price of good X, and p_Y is the market price for good Y.

The known, linear demand curve for good Y is $p_Y = a_Y - b_Y y$, where a_Y is the vertical intercept of the demand curve, and b_Y is the absolute value of the demand curve's slope. To capture the uncertainty in the demand for good X, suppose there are two states of the world. In the first state, State 1, which occurs with probability z_1, there is low demand for good X, and its linear demand curve is $p_X = a_X^L - b_X x$. Here a_X^L is the vertical intercept of the demand curve for good X given low demand, and b_X is the absolute value of the slope of good X's demand curve. Good X has high demand in State 2, which occurs with probability z_2, and the linear demand curve in this case is $p_X = a_X^H - b_X x$, where $a_X^H > a_X^L$. The slope of the demand curve for good X is assumed to have the same absolute value, b_X, in both State 1 and State 2. Thus, the uncertainty in the demand for good X is captured solely by a shift in the intercept of its demand curve from a_X^H in a period of high demand to a_X^L in a period of low demand.

Each good is produced with constant per unit cost. The constant marginal cost of good X is c_X, while c_Y is the constant marginal cost of good Y. Goods X and Y are assumed to be unrelated. Neither goods are substitutes or complements; therefore, a change in the price of one good does not affect the demand for the other good. In addition, there are no synergies in production as c_X and c_Y are unrelated.

Maximizing the Expected Utility of Income

The monopolist's profits in State 1, π_1, equals the sum of the certain profits from the sale of good Y and the profits from selling good X at the lower demand or

$$\pi_1 = (a_X^L - b_X x)x - c_X x + (a_Y - b_Y y)y - c_Y y \ . \tag{1}$$

Likewise, the profits the monopoly earns in State 2, π_2, equal the sum of the certain profits generated by the sale of good Y and the profits from the sale of good X with the increased demand or

$$\pi_2 = (a_X^H - b_X x)x - c_X x + (a_Y - b_Y y)y - c_Y y \ . \tag{2}$$

Given the uncertainty about the demand for good X, the monopolist determines the optimal values of x and y by maximizing its expected utility function, $E(U) = z_1U(\pi_1) + z_2U(\pi_2)$, where U(x) is the utility of income.[1]

Differentiating E(U) with respect to x and y results in two first-order conditions or

$$\partial E(U)/\partial x = z_1U'(\pi_1)(a_X^L - 2b_X x - c_x) + z_2U'(\pi_2)(a_X^H - 2b_X x - c_x) = 0 \tag{3}$$

and

$$\partial E(U)/\partial y = [z_1U'(\pi_1) + z_2U'(\pi_2)](a_Y - 2b_Y y - c_Y) = 0 \ . \tag{4}$$

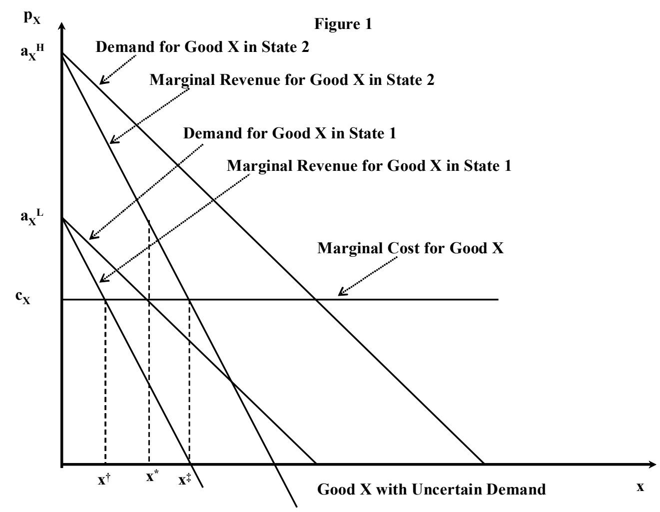
Since both z_1 and z_2 are positive, and the marginal utilities of income, $U'(\pi_1)$ and $U'(\pi_2)$, are positive, then equation (4) implies that $(a_Y - 2b_Y y - c_Y)$ equals 0. This means the multiproduct monopolist will produce that level of y that equates the marginal revenue of good Y with the marginal cost of good Y or

$$y^* = (a_Y - c_Y)/2b_Y \ . \tag{5}$$

Even though the demand for X is uncertain, the multiproduct monopolist produces the amount of Y that maximizes the profits associated with that known demand. The optimal amount of Y is the same regardless whether the demand for X is known with certainty or not. According to equation (5), the optimal level of Y is not a function of the marginal cost of producing good X. This result anticipates the comparative statics result below that shows the optimal level of y is invariant to changes in c_X.

Given the assumptions about the z_i and the $U'(\pi_i)$, i = 1, 2, equation (3) implies that the two terms - - $(a_X^L - 2b_X x - c_x)$ and $(a_X^H - 2b_X x - c_x)$ - - have opposite signs. Since $a_X^H > a_X^L$, it follows that $(a_X^L - 2b_X x - c_x) < 0$ and $(a_X^H - 2b_X x - c_x) > 0$. Given the uncertain demand for good X, the optimal level of x, x^*, must satisfy two conditions. First, at x^* the marginal revenue associated with State 1's lower demand for good X is less than the marginal cost of producing good

X. Conversely, the marginal revenue associated with the greater demand for good X in State 2 is greater than the marginal cost of producing good X at x^*.[2] If $x^\dagger$ is the profit-maximizing level of output of good X with lesser demand and $x^\ddagger$ is the profit-maximizing level of good X with the increased demand , then as Figure 1 shows, $x^\dagger < x^* < x^\ddagger$.[3] Thus, given uncertain demand for good X, the firm's optimal output of x exceeds the profit-maximizing amount in the case of the lesser demand, but is less than the profit-maximizing amount in the case of the greater demand.



THE SECOND-ORDER

Conditions of the Maximization Problem

The sufficient, second-order conditions of this optimization problem involve the second partial derivatives of the expected utility function, E(U). In matrix form, these second partial derivatives are

$$H = \begin{bmatrix} H_{11} & 0 \\ 0 & H_{22} \end{bmatrix} \tag{6}$$

where

$$H_{11} = z_1 U''(\pi_1)(a_X^L - 2b_X x - c_X)^2 + z_2 U''(\pi_2)(a_X^H - 2b_X x - c_X)^2 - 2b_X[z_1 U'(\pi_1) + z_2 U'(\pi_2)] \tag{7}$$

and

$$H_{22} = -2b_Y[z_1 U'(\pi_1) + z_2 U'(\pi_2)] \, . \tag{8}$$

Maximizing E(U) requires $H_{11} < 0$, $H_{22} < 0$, and the determinant of H, $\overline{H}$, which equals $H_{11}H_{22}$, must be positive. The off-diagonal terms of matrix H are zero because

$$[\partial^2 E(U)/\partial x \partial y] = [\partial^2 E(U)/\partial y \partial x] = [z_1 U''(\pi_1)(a_X^L - 2b_X x - c_X) + z_2 U''(\pi_2)(a_X^H - 2b_X x - c_X)](a_Y - 2b_Y y - c_Y),$$

and the last term in the above expression, $(a_Y - 2b_Y y - c_y)$, equals zero because of the first order condition in equation (4).

Comparative Statics: The Effect of Change in the Marginal Cost of Y

Evaluating equations (3) and (4) at the solutions, $x^* = x^*(c_X, c_Y)$ and $y^* = y^*(c_X, c_Y)$, and differentiating both of these equations with respect to c_Y, a standard comparative statics exercise finds

$$\begin{bmatrix} H_{11} & 0 \\ 0 & H_{22} \end{bmatrix} \begin{bmatrix} \partial x^* / \partial c_Y \\ \partial y^* / \partial c_Y \end{bmatrix} = \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} \tag{9}$$

where

$$\theta_1 = y[z_1 U''(\pi_1)(a_X^L - 2b_X x - c_X) + z_2 U''(\pi_2)(a_X^H - 2b_X x - c_X)] \tag{10}$$

and

$$\theta_2 = z_1 U'(\pi_1) + z_2 U'(\pi_2) > 0 \,. \tag{11}$$

Based on the assumptions of the model, θ_2 is unambiguously positive, as indicated, and the sign of θ_1 is indeterminate. Applying Cramer's rule to equation (9) obtains

$$\partial x^* / \partial c_Y = \overset{?}{\theta_1} \, \overset{-}{H_{22}} / \overset{+}{\bar{H}} \ \overset{>}{\underset{<}{=}} \ 0 \tag{12}$$

and

$$\partial y^* / \partial c_Y = \overset{+}{\theta_2} \, \overset{-}{H_{11}} / \overset{+}{\bar{H}} < 0 \,. \tag{13}$$

In the two equations above, the "+" signs appearing above certain terms indicates expressions that are unambiguously positive, just as the "-" signs appearing above other expressions denote terms that are unambiguously negative; however, the "?" sign above a term indicates an expression whose sign is indeterminate. The same convention in notation is followed below.

The economic interpretation of equation (13) is straightforward. Since the demand for good Y is known with certainty, if its per unit costs increases, an expected utility-maximizing, multiproduct monopolist will produce less of it. Since the demand for y is downward sloping, then $\partial p_Y^* / \partial c_Y > 0$. If c_Y increases, then a utility-maximizing, multiproduct monopolist will decrease y^*, and increase p_Y^*.

As mentioned above, the sign of the expression in equation (12) cannot be determined unless additional restrictions are placed on the utility function. θ_1 will be positive if, like Sandmo [11, p. 68], it is assumed that the utility function exhibits decreasing absolute Arrow-Pratt risk aversion. If $R_A(\pi)$ is the measure of absolute risk aversion, then $R_A(\pi) = -U''(\pi)/U'(\pi)$, where it is assumed $\partial R_A(\pi)/\partial\pi < 0$. Since $U'(\pi) = -U''(\pi)/R_A(\pi)$, the first order condition in equation (3) can be rearranged to obtain

$$0 = [z_1 U''(\pi_1)(a_X^L - 2b_X x - c_X)]/R_A(\pi_1) + [z_2 U''(\pi_2)(a_X^H - 2b_X x - c_X)]/R_A(\pi_2)\,. \tag{14}$$

Since $R_A(\pi_2) < R_A(\pi_1)$, then

$$0 < [z_1 U''(\pi_1)(a_X^L - 2b_X x - c_X) + z_2 U''(\pi_2)(a_X^H - 2b_X x - c_X)]/R_A(\pi_2) = \frac{\theta_1}{yR_A(\pi_2)}\,. \tag{15}$$

Since both y and $R_A(\pi_2)$ are positive, equation (15) ensures θ_1 is positive. With the additional assumption of decreasing absolute Arrow-Pratt risk aversion, then equation (12) becomes

$$\partial x^* \big/ \partial c_Y = \overset{+}{\theta_1}\, \overset{-}{H_{22}} / \overset{+}{\bar{H}} < 0\,. \tag{16}$$

According to equation (16), if the per unit cost of good Y - - the good whose demand is known with certainty - - increases, the monopolist will produce less of good X, the good with uncertain demand. Since the demand curve for good X is downward sloping, a rise in c_Y implies a fall in x^*, and an increase in p_X^*, or $\partial p_X^* \big/ \partial c_Y > 0$. In the case of uncertain demand, the multiproduct monopolist will cross subsidize increases in the cost of one good with increases in the price of another unrelated good. This outcome is intuitive as it expands Sandmo's [11] result. In determining the optimal amount of good X, the per unit cost of good Y acts like fixed cost, and in the presence of demand uncertainty, an increase in fixed cost leads to a decrease in x^*.

Comparative Statics: The Effect of Change in the Marginal Cost of X

To find the effect of a change in c_X on the optimal quantities of x and y, the first-order conditions in equations (3) and (4) are once again evaluated at the solutions, $x^* = x^*(c_X, c_Y)$ and $y^* = y^*(c_X, c_Y)$, and differentiated this time with respect to c_X. This comparative statics exercise results in the following two-equation system

$$\begin{bmatrix} H_{11} & 0 \\ 0 & H_{22} \end{bmatrix} \begin{bmatrix} \partial x^*/\partial c_X \\ \partial y^*/\partial c_X \end{bmatrix} = \begin{bmatrix} \theta_2 \\ 0 \end{bmatrix}. \tag{17}$$

Using Cramer's rule to solve for $\partial x^* / \partial c_X$, equation (18) shows that even in the case of uncertain demand, if the unit cost of good X increases, the monopolist will produce less of x, or

$$\partial x^* / \partial c_X = \overset{+}{\theta_2} \overset{-}{H_{22}} \Big/ \overset{+}{\bar{H}} < 0 \;. \tag{18}$$

Since the demand curve for good X is downward sloping, less x* means a higher p_X^*, or $\partial p_X^* / \partial c_X > 0$. Conversely, if the demand for Y is known with certainty, then equation (21) indicates the firm's optimal level of y^* is invariant to the value of c_X or

$$\partial y^* / \partial c_X = 0 \;. \tag{19}$$

This result also coincides with intuition. If the demand for god Y is known with certainty, why vary its output given a change in the cost per unit of good X? To do so would deviate from a known amount of output that maximizes profits in the market for good Y, and would reduce expected utility by more than the initial increase in c_X. Given no change in y^*, p_Y^*, remains constant, and $\partial p_Y^* / \partial c_X = 0$.

CONCLUSIONS

This paper investigates how a change in per unit cost of one good affects the prices of a multiproduct monopolist. Using a simple model of a firm producing two goods with linear demand curves and constant per unit costs, the case where one of the goods has uncertain demand is investigated. Demand uncertainty is captured by assuming two possible states of the world where the demand curve for good X shifts parallel to the left or right. When the demand curves are known with certainty, changes in the unit cost of one good do not affect the price of the other unrelated good. In this case, the profit-maximizing monopolist does not offset increases in the cost of one good by charging a higher price on the other good. When the demand for good X is uncertain and the firm exhibits decreasing absolute Arrow-Pratt risk aversion, an increase in the cost of good Y will lead the expected utility-maximizing monopolist to charge higher prices for both good X and good Y. So in the case of uncertainty, a monopolist will subsidize the higher cost for one good with a higher price for the other unrelated good, demonstrating a key point of the paper. The next key result of the paper shows that an increase in the cost of the good with uncertain demand will result in less of it being produced, but this increase in cost has no effect on the output and price of the good whose demand is known with certainty.

This simple model opens several avenues for future research. Instead of a simple discrete probability approach to model uncertainty, the model should be expanded to continuous probability to verify that its results are robust. Other forms of demand uncertainty rather than parallel shifts in the demand curve need to be investigated to also ensure the robustness of the results. Another extension of the model would be to disaggregate per unit cost into specific input prices and to examine how changes in input prices affect the prices charged by a multiproduct firm. These inputs could be common or unique to each product. But this simple model finds one key result. With uncertain demand, it is possible that increases in the cost of an unrelated product may lead a monopolist to charge higher prices for its other products.

ENDNOTES

1. Clearly, $0 < z_1 < 1$, $0 < z_2 < 1$, and $z_1 + z_2 = 1$. Additionally, the marginal utility of income, $U'(\pi_i)$, is assumed to be positive for i = 1 or 2. Risk aversion would imply $U''(\pi_i) < 0$ for i = 1 or 2.
2. Given the demand for good X in State 1, the marginal revenue equals $a_X^L - 2b_X x$, and the marginal revenue of good X in State 2 is $a_X^H - 2b_X x$.
3. At $x^{\dagger}$, $a_X^L - 2b_X x = c_X$, while $a_X^H - 2b_X x = c_X$ at $x^{\ddagger}$.

REFERENCES

Arrow, K. J. 1965. *Aspects of the Theory of Risk-Bearing*. Helsinki: Yrjö Hahnsson Foundation.

Bernhardt, Irwin. 1972. 'Fixed Costs' and the Competitive Firm Under Price Uncertainty: Comment. *The American Economic Review* 62(1/2): 193.

Brick, Ivan E. and Harsharanjeet Jagpal. 1981. Monopoly Price-Advertising Decision-Making Under Uncertainty. *Journal of Industrial Economics* 29(3): 279-285.

Dhrymes, Phoebus. 1964. On the Theory of the Monopolistic Multiproduct Firm Under Uncertainty. *International Economic Review* 5(3): 239-256.

Harris, Milton and Artur Raviv. 1981. A Theory of Monopoly Pricing Schemes with Demand Uncertainty. *The American Economic Review* 71(3): 347-365.

Leland, Hayne E. 1972. Theory of the Firm Facing Uncertain Demand. *The American Economic Review* 62(3): 278-291.

Meyer, Robert A., "Monopoly Pricing and Capacity Choice Under Uncertainty." *The American Economic Review* 65(3), 1975, 326-337.

——— . 1976. Risk-Efficient Monopoly Pricing for the Multiproduct Firm. *Quarterly Journal of Economics* 90(3): 461-474.

Mills, Edwin S. 1959. Uncertainty and Price Theory. *Quarterly Journal of Economics* 73(1): 116 – 130.

Pratt, J. W. 1964. Risk Aversion in the Small and in the Large. *Econometrica* 32(1/2): 122-136.

Sandmo, Agnar. 1971. On the Theory of the Competitive Firm Under Price Uncertainty. *The American Economic Review* 61(1): 65 – 73.

Sandmo, Agnar. 1972. 'Fixed Costs' and the Competitive Firm Under Price Uncertainty: Reply. *The American Economic Review* 62(1/2): 194-195.

Zabel, E. 1970. Monopoly and Uncertainty. *The Review of Economic Studies* 37(2): 205 – 219.

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