TESTING MULTIPLE FINANCIAL BUBBLES IN THE NASDAQ INDEX

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ABSTRACT

Identifying and dating financial bubbles in real time is in the forefront of current empirical research. But their complex nonlinear structure makes the econometric testing challenging, to say the least. A new recursive flexible window methodology has been provided by Phillips, Shi and Yu (2015), which gives consistent results and delivers significant power gains when multiple bubbles occur. It successfully identifies the well-known historical episodes of exuberance and collapse. This is possibly the first application of this statistical procedure to the entire NASDAQ index period (1971-2018). The results are very encouraging indicate evidence in favor of multiple upheavals, which correspond closely to reality. **JEL Classification**: F3

INTRODUCTION

In the economics literature there are have multiple tests to detect ex-post the crisis and then explain it. But there was no test to ex-ante identify the origination of a bubble which is in the making. There were no econometric detectors of a future market crisis. Experts have often said that the present crisis was preceded by "asset market bubbles" and / or "excessive credit expansion" but the fact of the matter remains that there are no have good quantitative markers which can ex-ante indicate the genesis of a crisis, which may lead to a catastrophe down the line. If there were quantitative observable "warning signs" many an economic debacle can be avoided. After the most recent global financial crisis of 2007-2009, the main thrust in the Basel III accord was to emphasize on more close and determinate market surveillance, so that bankers and policy makers could be forewarned of a possible impending implosion.

Thus the task at hand is to try to identify possible quantitative markers from the data, that something is "awry" and that a speculative bubble is probably taking shape. It will worsen if measures to "quell" it is not taken, now. There were no econometric detectors of a future market crisis till Phillips, Wu and Yu (PWY henceforth, 2011) presented a recursive method to detect exuberance in asset prices specially during an inflationary phase. The advantage here being that the early detection (ex-ante acknowledgement) can help banks / regulators / policy makers to address the problem in its nascent state. PWY was very effective in the early detection of bubble markers, provided there was a single bubble / turbulence in the data sample. They proved the effectiveness of the test using NASDAQ data in the 1990's (PWY, 2011) and the US housing bubble (PY, 2011). This was an incredible contribution to the economics research literature.

But then came the question of "economic reality" which showed that there usually were multiple recurring financial crises, over long periods. Ahmed (2009) gave us evidence of 60 different financial crises, in the 17th century alone. Thus the next step in the evolution of these detection tests was to create the one that could identify multiple bubbles in the same sample period. A test to clearly make periodic collapsing and recovering economic data was simply not there. This recursive identification is extremely complex compared to identifying a single bubble. The main problem is computationally handling the non-linear structure of multiple breaks / bubbles in the data. With the presence of multiple break points, the discriminatory power of the detectors go down dramatically and hence the upswings and downswings are not decipherable in the same data stream.

Thus the challenge is twofold:

1) Come up with a statistical metric which can detect multiple factual fractures in the non-linear data stream, and

2) Be powerful and effective enough so as not to have a low false negative detection tolerance (to avoid unnecessary policies) and also a high positive detection tolerance (so as to ensure good and early effective policy application.)

This challenge was met head on by Phillips, Shi and Yu (PSY henceforth) in their 2014 paper, which offers the first powerful and credible "quantitative metric" to detect multiple bubbles / upheavals / exuberance and bandwagon effects in both directions in financial data. It was the next developmental step in this research progression. This test is based on a sequence of forward recursive right tailed ADF unit root tests, using the Sup ADF (designated SADF) measure. This process allows for a dating strategy to identify the origination and termination dates of a specific bubble. This is achieved by using "backward regression techniques." This detection algorithm is better able to date the ups and downs of financial data, as opposed to the CHOW tests, CUSUM tests etc. as evidenced by Homm and Breitung (2012). Its added strength is that it can detect exuberance in the data arising from different sources, as would happen in real life. It is an extension of the SADF tests, in form of a generalized SADF called the GSADF method. It includes a recursive backward regression technique, to time identify the origin and collapse of bubbles. It is a right tailed ADF test, but has a flexible window width to separate one bubble from the next, to the next sequentially, since their lengths are bound to be different. In structure and logic, it is analogous to the left-sided recursive unit root test of Leybourne, Kim and Taylor (2007), this being a right-sided double recursive unit root test.

This test is able to detect different start and end points of bubbles in real time data, i.e., identify and separate multiple bubble episodes over the same sample set. This test has been proven to consistently give good results, when multiple bubbles are present. Thus it can credibly be applied to analyzing long term historical data. Along with the ex-ante dating algorithm and the GSADF test, the authors develop a modified PWY algorithm, which reinitializes the test sequentially, after the detection of each bubble. This sequential test works in deciphering multiple bubbles from explosion to collapse, and separate them over time. PSY 2014 apply it to the S&P 500 stock market data from January 1871- December 2010. It has been able to identify all the historically documented bubble episodes, like the 1929 crash, 1954 boom, 1987 black

Monday and the latest dot-com bubble.

This paper does an early (and quite possibly the first) application of the PSY recursive flexible window methodology, to test for the presence of multiple bubbles in the entire NASDAQ data set. PWY 2011, examined for a single bubble in the NASDAQ data in the 1990's, going up-to 2005. This paper goes on to extend their work by examining for the presence of multiple bubbles (as opposed to a single bubble as in PWY 2011) in the entire NASDAQ data spanning from March 1971 to February 2018. It also covers the volatile financial period from 2005 to 2017, including the 2008-2009 crisis. Section 2 describes the reduced form model, the new rolling window recursive test and its limit theory, given in PSY (2015, 2015 b). Section 3 elaborates the data stamping strategies, to separate single, double and multiple bubbles in the same sample period. Section 4 is simulation results of the size, power and performance of the dating strategy tests. In section 5, the paper applies the PWY test, the sequential PWY test and the CUSUM test to the entire NASDAQ data set. Section 6 concludes.

ROLLING WINDOW TEST FOR BUBBLES

It originates with the standard asset pricing model:

$$P_t = \sum_{i=0}^{\infty}\left(\frac{1}{1+r_f}\right)i\, E_t(D_{t+i} + U_{t+i}) + B_t \qquad (1)$$

where
P_t = after dividend price of an asset
D_t = payoff (dividend) from the asset
r_f = risk free interest rate
U_t = unobservable fundamentals
B_t = bubble component

Here $P^f_t = P_t - B_t$ (market fundamentals) and B_t satisfies the sub martingale property

$$E_t = (B_{t+1}) = (1+r_f)B_t \qquad (2)$$

This equation sets up the alternative scenarios for the presence / absence of bubbles in the data. For example: If there are no bubbles, the $B_t = 0$, then the degree of non-stationarity [I(0) or I(1)] of asset prices is controlled by asset payoffs or dividends (D_t) and the unobservable economic / market fundamentals.

According to Phillips and Magdalinos (2007), explosive behavior in asset prices is a primary indicator of market exuberance, which can be identified in empirical tests using the "recursive testing procedure" like the right side unit root test of PWY. This recursive procedure starts with a martingale null (with drift to capture long historical trends in asset data.) The model specification is:

$$y_t = dT^{-n} + \theta_{yt-1} + \epsilon_t \qquad (3)$$

where ϵ_t is iid $(0, \sigma^2)$, $\Theta = 1$, and d is a constant, T is the sample size, and the parameter

η controls the magnitude of the intercept and the drift, as $T \rightarrow \infty$. Solving eq. 3, gives us the deterministic trend, dt/T^n. Here there are three possibilities:
1) If n > 0, the drift will be small compared to the linear trend.
2) If n >1/2, the drift is small relative to the martingale
3) If n = 1/2, the output behaves like a Brownian motion, which is evident in many financial time series data.

The researcher needs to be careful and exercise caution because all types of model specifications are sensitive to intercepts, trends and trend breaks etc. as described in PSY (2014). Eq. 3 is tested for exuberance using the rolling window ADF approach or the recursive approach of the authors. The basic logic is that if the rolling window regression starts from the r_1^{th} fraction and ends with the r_2^{th} fraction (from sample size T), then $r_2 = r_1 + r_w$, where r_w is the size of the window. This model is:

$$\Delta y_t = \propto_{r1,r2} + \beta_{r1,r2}\ y_{t-1} + \Sigma^k_{i=1}\ \gamma^i_{r1,r2}\ \Delta_{yt-1} + \epsilon_t \qquad (4)$$

where k is the lag length, and is iid, with $(0, \sigma^2_{r1,r2})$. This specification is reformulated to include the presence of "multiple bubbles" to separate the market switching time periods from explosion to contraction, and again explosion sequentially. They use the Sup ADF test called SADF. It is a recursive / repeated estimation procedure with window size r_w., where r_w goes from r_0 (smallest sample window fraction) to r_1 (largest sample window fraction), and sample end point $r_2 = r_w$, going from 0 to 1. The SADF statistic i: (1)

$$\text{SADF } (r_0)\ n = \sup_{r2 \in [r0,1]} \text{ADF}^{r2}_0$$

The ADF regression is run on eq. 4, recursively, but continuously on sub-samples of the data based on window width chosen according to $r_0, r_1, r_2 \ldots\ldots r_w$. The subsamples chosen here are more extensive than the SADF test. The difference here is that tests used in this paper allow the window width to change within the feasible range where $r_w = r_2 - r_1$. The GSADF statistic is:

$$\text{GSADF } (r_0) = \sup r_2 \in [r_0, 1]\ \{\text{ADF}^{r2}_{r1}\} \qquad r_1 \in [0, r_2 - r_0] \qquad (5)$$

The GSADF statistic as given in eq. (5) (2). Here it can be seen that the limit distribution of the GSADF holds (is identical), but with the intercept and the assumption of a random walk structure, there will be no drift or small drift. The GSADF's asymptotic distribution depends on the "smallest window width size r_0." Care needs to be exercised on choosing the width of r_0. It depends on the number of observations in the sample. The empirical steps are:
1) Determine ADFr_2 and the sup ADF within the feasible range of r_2 (from r_0 to r_1.) The origination of the bubble is dated. This procedure imposes the condition that the bubble marker is the existence of a critical value greater than L_T = Log (T). This separates the short and temporary market blips (which happen all the time in real life) from actual exuberance. Dating is done using the formula. (3, 4, 5)

$$r_e = inf_{r2\in[r0,1]} \{r_2 : ADF_{r2} > cv^{\beta T}_{r2}\} \quad (6)$$

and

$$r^{\wedge}_f = inf_{r2\in[r^{\wedge}e + \log(T) / T,1]} \{r_2 : ADF_{r2} > cv^{\beta T}_{r2}\} \quad (7)$$

where $cv^{\beta T}_{r2}$ is the 100(1-β_T) % critical value of the ADF statistic based on $[T_{r2}]$ observations. Here $\beta_T \to 0$, as $T \to \infty$.

DATA STAMPING STRATEGIES

The idea is to identify bubbles in real time data and then look for the "markers" identifying those bubbles / episodes of market exuberance. The problem is that the standard ADF test can identify extreme observations, as r = $[T_r]$, but cannot separate between a bubble phase observation from one which is part of a natural growth trajectory. Market growth is not an indication of bubbles. Thus ADF tests may result in finding "pseudo bubble detection." Making this distinction is the major contribution of this test. The authors run backward sup ADF or backward SADF tests, to improve the chances of deciphering a bubble from a growth trajectory. The recursive test means running SADF backwards on the sample, increasing the sample sequence using a fixed sample r_2, but varying the initial point from 0 to $(r_2\text{-}r_0)$. This gives the SADF statistic: $\{ADF^{r2}_{r1}\} \in |0, r_2 -0|$

Bubbles are inferred from the backward SADF statistic or the BSADF $r_2(r_0)$. The origin of the bubbles, the date and timing is the first observation whose BSADF statistic exceeds the critical value of the BSADF. The bubble ending date / time frame is the first observation whose BSADF is below the BSADF critical value. The intermediary time frame is the duration of the bubble. The origination / termination dates are calculated thus:

$$r^{\wedge}_e = inf_{r2\in[r0,1]} \{r_2 : BSADF_{r2}(r_0) > scv^{\beta T}_{r2} \quad (8)$$

$$r^{\wedge}_f = r_{2\in}[inf_{r^{\wedge}e + \partial \log(T) / T,1]} \{r_2 : BSADF_{r2} (r_0) > scv^{\beta T}_{r2} \quad (9)$$

where $scv^{\beta T}_{r2}$ is the 100(1-β_T)% critical value of the sup ADF statistic, based on $[T_{r2}]$ observations. β_T goes to zero, as the sample size approaches infinity. The distinction between the SADF and the GSADF (backward sup ADF) tests, both run over $r_2 \in [r_0,1]$ is given by the statistic as:

$SADF (r_0) = sup_{r2\in[r0,1]} \{ADF_{r2}\}$
and
$GSADF (r_0) = sup_{r2\in[r0,1]} \{BSADF_{r2}(r_0)\}$

SIMULATIONS

Simulations were performed to examine the credibility of the PWY, sequential PWY, CUSUM and the GSADF test, in terms of size and power, but most importantly their capability to identify multiple bubble episodes. The basic data generating process is given by:

$$y_t = dT^{-n} + \theta_{yt-1} + \in_t \qquad (10)$$

with δ =n =1. They examine two different models, namely Evans (1991) collapsing bubble and the PWY model. Simulations using the same data set, same number of observations / replications show that the size distortion of SADF > GSADF. The next question is the effect of the lag selection length. Both SADF and the GSADF have size distortion weakness. But its magnitude is small, when a fixed lag length is used in recursive tests. But GSADF has smaller distortion than SADF and thus has a leg up on the latter in lowering the probability of "false detection." The authors (PSY: 2014, 2015 and 2015, b) recommend the fixed lag length use with the GSADF test for multiple bubbles. They find that the SADF test has an inherent weakness, evidenced again and again. It could not identify bubbles when the full sample was used, but was able to do so when the sample was truncated. But the recursive application of the GSADF test was able to identify multiple bubbles, without having to arbitrarily truncate / segment / re-select sample starting points. This is a major advantage of GSADF over the SADF procedure. Moreover, the results show that the bubble identification power of the GSADF test increases as the sample size increases.

EMPIRICAL APPLICATIONS

This paper uses monthly data for the NASDAQ for the period March 1971 to February 2018, for a total of 564 observations. This data set was obtained from DataStream. [(6)] The data used is the NASDAQ stock price index for the relevant month. The SADF and the GSADF tests are applied on the stock price index according to the basic model in eq. (1). The results are given in table 1. Also given are the critical values of the two tests obtained from 2000 replications of the 564 observations.

Both tests find evidence of bubbles or explosive sub-periods over the long-term data (test statistics exceed the critical values). This paper then conducts a bubble monitoring exercise for the NASDAQ stock market using the backward SADF test and its critical value in Figure 1 (using the PSY strategy), and the backward ADF statistic and its critical value in Figure 2 (using the PWY strategy).

In Figure 1, the existence of a bubble, test statistic greater than the critical value, is evident in the late 1970s and early 1980s, which corresponds with the recession during that period, and then again from 1985 to 1987. There is a sharp dip in the market right around the time of the "Black Monday" (October 19, 1987) crash. Most of the 1990s shows evidence of a bubble. The drop off in 2000 corresponds with the bursting of the technology bubble. There is no evidence of a bubble in the first decade of the 21st century, including during 2007-09 (financial crisis) which is rather surprising. There is however evidence of another bubble starting in 2016-17. This corresponds with the run up in stock prices that has occurred since November 2016.

Figure 2 shows the bubble monitoring exercise using the backward ADF test statistic from PWY paper. The conclusions are almost the same except that Figure 2 does not show a bubble at the end of the 1970s. This indicates the existence of multiple bubbles in the extended data set which corresponds to period of recession and expansion, but does not (surprisingly) include the financial crisis of 2008-09.

CONCLUSION

The new test, the GSADF procedure is a recursive test, able to detect multiple bubbles. It's a rolling window, right sided ADF unit root test, with a double sup-window selection criterion. The SADF test is good, but it cannot credibly detect multiple bubbles over the same sample data set. The GSADF test overcomes this weakness and has significant discriminatory power in detecting multiple bubbles, making it very relevant in studying the "time trajectory" of long historical data sets. The availability of the data is an issue, plus using a technology weighted index might skew the results. An extension of this study would be to apply this methodology to a wider variety of stock indices from a number of different countries.

NOTES

1 Then there is the Markov-switching test of Hall, et.al (1999), to detect explosive behavior in the data sample, but it is open to suspicion since Shi (2013) found it to be susceptible to "false detection of explosiveness." Also, according to Funke et.al. (1994) and Van Norden and Vigfusson (1998), general filtering algorithms cannot differentiate between spurious explosiveness (the marker being high variance) as opposed to generic explosive behavior. The general approach of SADF is also used by Busetti and Taylor (2004) and Kim (2000) among others, to study "market bubbles" but the simulation study done by Homm and Breitung (2012) finds the PWY (SADF) test to be the most powerful metric in detecting multiple bubbles.

2 Eq. (5), Theorem 1, from PSY (2014)

3 The data process before the origination of the bubble is assumed to be a random walk for convenience, and it is the usual practice, but not necessary for the asymptotic properties to hold.

4 The authors have proven the consistency of ($r^{\wedge}_{e}$, $r^{\wedge}_{f}$) in PY (2009).

5 This sequential procedure (for proper and credible application) requires a long set of observations, the longer the better, in order to re-initialize the test process after a bubble.

6 DataStream proprietary data was purchased from EIKON, made possible due to a research grant of Professor Dutt, from the Richards College of Business, University of West Georgia. The data are taken from DataStream International. We collect monthly observations on the Nasdaq composite price index (without dividends) and the Nasdaq composite dividend yields, and compute the Nasdaq composite dividend series from these two series. We use the Consumer Price Index (CPI), which is obtained from the Federal Reserve Bank of St. Louis, to convert nominal series to real series.

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TABLE 1

	Test Statistic	Finite Sample Critical Values		
		90%	95%	99%
SADF	10.4860	1.2084	1.5316	2.1665
GSADF	10.4860	1.9805	2.2458	2.8082

FIGURE 1

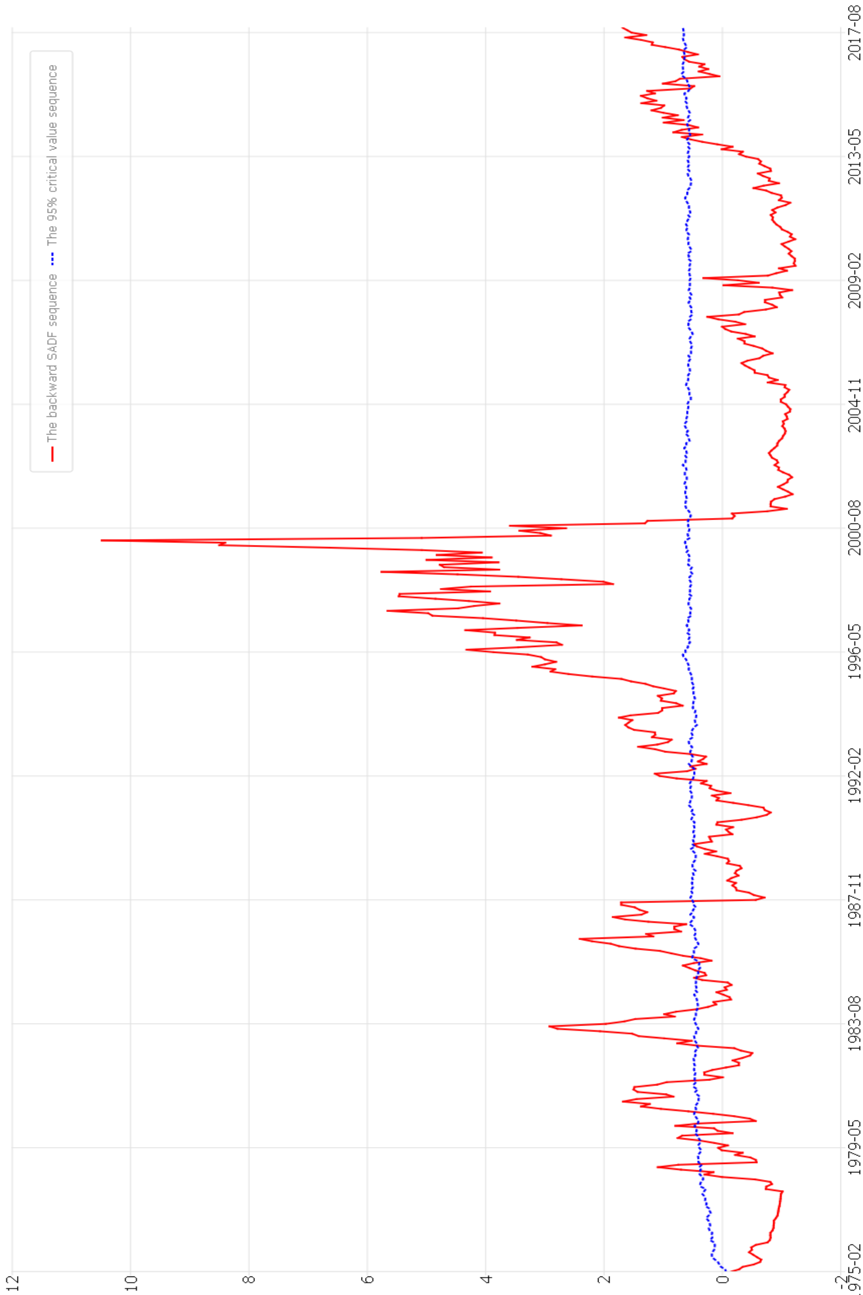
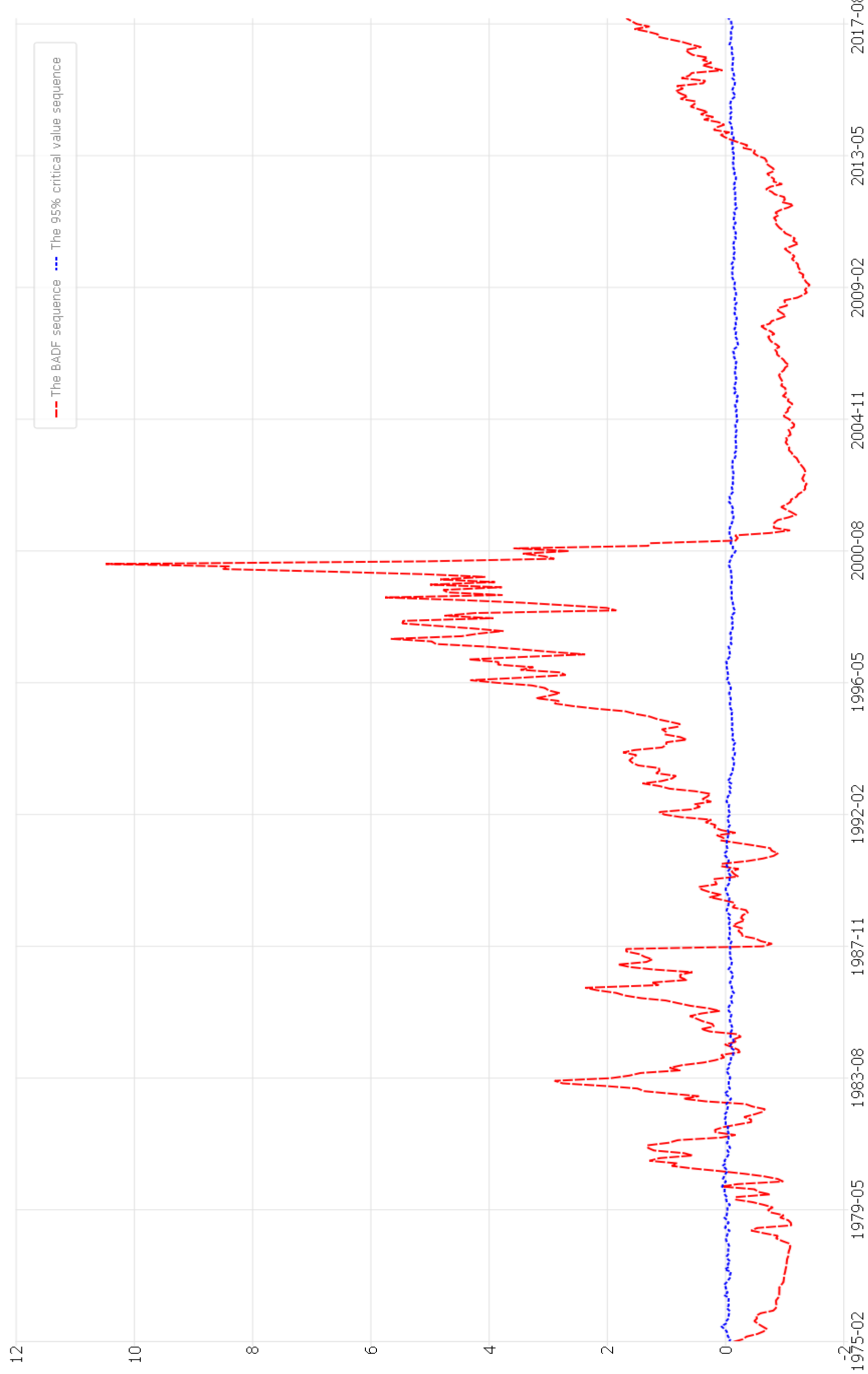

The backward SADF sequence
The 95% critical value sequence
12
10
8
6
4
2
0
-2
1975-02
1979-05
1983-08
1987-11
1992-02
1996-05
2000-08
2004-11
2009-02
2013-05
2017-08

FIGURE 2



The BADF sequence
The 95% critical value sequence
12
10
8
6
4
2
0
-2
1975-02
1979-05
1983-08
1987-11
1992-02
1996-05
2000-08
2004-11
2009-02
2013-05
2017-08