# *THE RATIONALITY ASSUMPTION, LEARNING AND UTILITY ATTAINMENT OVER TIME*

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**ABSTRACT**

The degree to which economic decision makers behave "rationally" and the related issues of applicability and usefulness of neoclassical utility maximization theory have long been subjects of debate within the economics profession. With the emergence and ascendency of the behavioral and experimental fields of economics over the last decade, that debate has intensified. Absent perfect knowledge (information and calculating skills) by decision makers, one would expect a "gap" between maximum utility attainable from a decision and actual utility realized. But with experience and learning, one would expect the magnitude of that gap to diminish. This paper presents a model in which the individual is viewed as going through an iterative process through which he/she learns by making choices and evaluating the effects of those choices on utility attainment. **JEL Classifications:** D01, D03

**INTRODUCTION**

If one were assigned the task of identifying the one constancy that most singularly characterizes the history of economic thought, a likely candidate for that distinction would undoubtedly be the discipline's tradition of diversity of opinion (theory) and debate. That has certainly been true in the case of neoclassical utility maximization theory. And while modern behavioral economics, as it has developed over the last couple of decades, is by no means the first challenge to the efficacy of that body of theory, it clearly is the most widely recognized, utilized and championed today. Perhaps the position of many, if not most, of today's behavioral economists was expressed aptly by The Economist on the event of Vernon Smith and Daniel Kahneman's Nobel recognition in 2002: "Bid farewell to the cold-hearted humans that, since Adam Smith's day, economists have used as their models…. Now meet the new, sensitive homo economicus: he…is more laid back, relying on intuition and rules of thumb to make decisions, often without perfect knowledge."

The degree to which economic decision makers behave "rationally" and the related issues of applicability and usefulness of neoclassical utility maximization theory have long been subjects of debate within the economics profession. With the emergence and ascendency of the behavioral and experimental fields of economics over the last decade or so, that debate has intensified significantly. Absent perfect knowledge (information and calculating skills) by decision makers, one would expect a "gap" between maximum utility attainable from a decision and actual utility realized. But with experience and learning, will that gap diminish? The purpose of this paper is twofold: First, we provide a brief glimpse into the nature of, and (our assessment of) the current state of, the traditionalist-behaviorist debate. And second, we present a model in which the individual is viewed as going through an iterative process through which he/she learns by making choices and evaluating the effects of those choices on utility attainment.

## THE DEBATE

### Behaviorists' Position

Over the decade that has passed since the behavioral/experimental economics revolution made that first big public splash via Nobel, much ink has been devoted to convincing economists (and non-economists alike) that (1) traditional economists routinely use models that are grossly inconsistent with the findings of psychology, (2) traditional economics relies far too heavily on the fatally flawed assumption that humans act rationally, and that (3) while these theories, based as they are on wildly unrealistic assumptions, sometimes produce useful predictive results, there is a strong need to reunify economics and psychology.[1] Colin Camerer (1999), a leading behavioral economist, was making these very arguments well before Nobel took note of the growing schism within the discipline: "Because economics is the science of how resources are allocated by individuals and by collective institutions like firms and markets, the psychology of individual behavior should underlie and inform economics, much as physics informs chemistry; archaeology informs anthropology; or neuroscience informs cognitive psychology."

Relative to these general observations and arguments, there certainly has been no dearth of experimental evidence suggesting that human decision makers, influenced by forces and practices such as frames, loss aversion, mental accounts, hyperbolic discounting, cues and endowment effects, exhibit tendencies of "situationalism" at least as often as they exhibit the non-emotional, calculating finesse suggested by a strict application of the traditional utility maximization model.[2]

### Traditionalists' Response

In response to criticisms cast at traditional economic theory by experimental and behavioral economists, proponents of the former approach argue that laboratory type experiments do not have the same force, validity and applicability when applied to human behavior. Their position is that in the world of the "hard" sciences, what is true in the lab may (at least generally) be true for the world outside the laboratory as well, but that when economists and psychologists try to use experiments to explain human behavior, that link tends to break down. Experiments have unquestionably become an important part of the discipline. However, whether the economist conducts his/her experiments in the laboratory or in the field, it is still experimentation and thus subject to the adulterating effects of the process. As the great theoretical physicist Werner Heisenberg (1958), noted, "[T]he measuring device has been constructed by the observer, and what we have to remember is that what we observe is not nature itself, but nature exposed to our method of testing." If the quantum physicist cannot eliminate the uncertain impact introduced by his/her testing methodology, how likely is it that the economist or psychologist can do better?

Perhaps more than anyone else, John List has demonstrated that many of the criticisms leveled at traditional theory by behavioral/experimental economists have been a reflection of flawed experimental design as much as, or more than, of flawed theory. The facts that (1) subjects may know they are being watched, (2) there exists the possibility for experiments to be controlled—whether consciously or unconsciously—by those conducting the experiments, (3) the stakes, in terms of costs, consequences, rewards or benefits, tend to be small, and (4) there is often the ability for subjects to be self-selecting, all call into question conclusions drawn from such experiments. Additionally, we would note that experiments rarely, if ever, provide sufficient time and/or repetitions to capture the effects of experience and learning on subject decision making. All too often, the experiments take on the characteristics of a "snapshot in time" as opposed to being recognized as merely a part of a dynamic process. Thus, despite the usefulness of experiments, they are not without limitations. As Gary Becker has argued tirelessly for years, "There is a heck of a difference between demonstrating something in a laboratory, in experiments, even highly sophisticated experiments, and showing that they are important in the marketplace." (Clement, 2002, p. 9)

Two other reservations often expressed by "traditional" economists relative to the claims of behavioral economists have to do with issues of aggregation and the effects of experience and learning in the marketplace. Consider, for examples, the following observations by Becker and Edward Glaeser. Becker, who has expressed this argument in many forms and forums, asserts: "A market economy is a group of specialists who are integrated by exchange. It may be that each of these specialists is terrible at other activities, but the whole aggregate can be highly efficient. The aggregate may make few mistakes. One of the things some behaviorists have missed is that a specialized economy eliminates many mistakes because vulnerable people don't get put into positions where they can make these mistakes." (Clement, 2002, p. 9) Edward Glaeser, a University of Chicago trained, Harvard economist, has expressed it thus: "The great achievement of economics is understanding aggregation….Our discipline has always been about the wealth of nations, not individuals." Relative to the behaviorists, he argues "Much of the early work has focused on changing the core of economics with work on individuals. It's hard to read the bulk of research and not think it specializes more on individuals." (Stewart, 2002, p. 5) Interestingly, while behavioral economists may have been slow to recognize fully the implications of the individual-versus-aggregate dichotomy, mathematicians understand it well. Steven Strogatz (2012), an applied mathematician at Cornell, observes, "Things that seem random and unpredictable when viewed in isolation often turn out to be lawful and predictable when viewed in the aggregate."

The confluence of aggregation, learning and experience, and the market represents a strong force. Again, as expressed by Becker: Barnum said there's a sucker born every minute of the day. Well, suckers lose their money. Another example: I have fair dice, but you believe that a 12 is going to come up half the time. I would love to play craps against you. You will continue to lose until either you change your beliefs, or you lose your shirt. Exchange and the division of labor do not eliminate all the issues brought up by behavioral economics but I believe 'behavioral' economics has a different place in modern economies than is often claimed."(Clement, p. 10) Notwithstanding these arguments, we do not mean to argue that there are not significant implications of the research findings of behavioral economists that extend well beyond the individual.

**Our Notion**

Few if any "traditional" economists would argue that consumers, producers and even governments do not at times make bad economic decisions based on seemingly "irrational" attention paid to sunk costs. Nor would they argue that consumers never make "irrational" decisions as a result of "framing" influences (a la Tversky and Kahneman prospect theory). It is also well accepted that some taxpayers exhibit a negative time preference for money, opting to inflate withholdings in order to receive a larger tax refund. Many would accept the notion, generally attributed to Richard Thaler, that an "endowment effect" (or "divestiture aversion") may enter into consumer choice at times, and in a fashion considered inconsistent with the tenets of traditional economic theory. It is easy to make a case that many of us at times apply unreasonably high (or "irrational") discounts rates when considering future costs and/or consequences of saving for "rainy days" or retirement versus current consumption, studying for exams versus enjoying that great party, etc. Additionally, it cannot be denied that individuals routinely engage in selfless acts of altruism—which behavior some argue is also inconsistent with the tenets of traditional, utility-maximizing theory.

Do these observations, however, and their associated, apparent evidence of departure from the principles of rational decision making and/or expected utility maximization diminish, or even destroy, the efficacy and usefulness of basic neoclassical theory? The ubiquitous obituaries heralding (and often seemingly celebrating) the death of utility maximization theory bring to mind three earlier quotes that seem apropos. First, as argued more than half a century ago by another noted (and Nobel honored) economist, Paul Samuelson (1951): "In economics it takes a theory to kill a theory; facts can only dent the theorist's hide." Second, the famous disclaimer of Mark Twain comes to mind: "The reports of my death are greatly exaggerated." And finally, in responding to a young critic of his sales maximization model, William J. Baumol (1964) advised: "[His critic] should recognize that, in order for a writer to produce something which is original and correct, it is not absolutely necessary that his predecessors have been wrong."

We argue that (a) ignorance does not equal irrationality. The fact that people do not always make choices that are in their own interest may simply be a result of their having imperfect information and/or their having incorrect theories about the world, (b) being unable to see an individual's utility function may lead the experimenter/observer to reach incorrect conclusions when attempting to evaluate decisions, and (c) that still the ultimate test, and therefore the ultimate answer, to the question depends on whether there exists—or the critics develop— an alternative, superior theory that yields better and more compelling explanatory power and predictive results. To date, we tend to agree with Pesendorfer (2006), who asserts that "…behavioral economics remains a discipline that is organized around the failures [or perceived failures] of standard economics. The typical contribution starts with a demonstration of a failure of some common economic assumption (usually in some experiment) and proceeds to provide a psychological explanation for that failure." And while we do not argue that experimental and/or behavioral economics is without value, relevance and/or use, we do argue that the behaviors observed and described by such economists, or psychologists, may provide an interesting (to some, especially psychologists) glimpse inside the "black box" of decision making, but they routinely fail to "beat" expected utility theory in terms of yielding "valid and meaningful predictions about phenomena not yet observed."

### Allowing for Learning

Today, the notion of a perfectly impassionate, omniscient decision-making "homo economicus" serves perhaps primarily as a straw man used by critics of neoclassical economic analysis to lend added import to their experiments. It is difficult to point to a serious economist in the last hundred years who has held to such a strait-jacket view of individual decision making. As noted by Arthur (1994) several years ago, "Economists have long been uneasy with the assumption of perfect, deductive rationality in decision contexts that are complicated and potentially ill-defined." For example, consider the arguments of Roy Harrod (1939) in the "marginalism" debates that emerged in the late 1930s and early 1940s relative to the issue of traditional profit-maximization theory. Following several studies that concluded that, empirically, it was not evident that entrepreneurs followed the marginalist principles of profit-maximizing, cost-minimizing behavior in running their firms, Harrod put forward the concept of "groping." Essentially, Harrod's response to the criticism that profit-maximizing behavior was not observed in many firms was that decisions would nevertheless be made and there would be a "natural selection" process whereby profit-maximizing decisions would be rewarded and inferior decisions penalized. He argued that while entrepreneurs may not be able to engage in profit-maximizing decisions with perfect information and/or certainty, they will "grope" for it, i.e., they will learn and adjust. Similarly, Boulding's (1966) concept of "learning through disappointments" represents an explicit representation that a learning process takes place in human decision making. We note that various critics of the efficient market hypothesis have discounted the theory on the basis that some decision makers over-react to new information (e.g., DeBondt and Thayer, 1985), while others have found just the opposite—underreaction rather than overreaction (e.g., Shleifer, 2000). What we find most significant, however, is that the decision makers do react to new information. They too grope. They learn.

As illustrated by the above references, economists have long acknowledged the existence, importance and impact of imperfect knowledge and learning. More recently, Drew Fudenberg, David Kreps, David Levine, and others have greatly expanded the analytical bounds of learning theory, employing sophisticated use of game theory involving Nash equilibrium concepts plus enhancements (and weakenings) such as self-confirming- and approximate equilibrium concepts.[3] Even with that, however, David Levine (2012), a self-described behavioral economist, acknowledges, "…behavioral economists, psychologists, economists and computer scientists model human learning by what can only be described as naïve and primitive models. Some of these models have various errors and biases built in. Even those models designed by computer scientists to make the best possible decisions cannot come close to the learning ability of the average human child—indeed, it is questionable that these models learn as well as the average chimpanzee or even rat."

The model presented in this paper is a more generalized one, asserting that learning occurs over time as the individual makes choices and evaluates the effect of those choices on utility attainment. The individual will make future choices based on the knowledge gained from these past choices. Additionally, the individual will observe the experiences of others and may even engage in formal education activities as well in order to gain knowledge about the structure of his environment. The individual will attempt to maximize utility attainment over his time horizon, given his stock of knowledge. We assert that the individual faces a learning curve, which implies that outcomes of choices do not always and automatically result in the attainment of the maximum possible utility. This fact in no sense invalidates, or even significantly reduces the value of, the assertion that individuals attempt to maximize utility when making choices. The operative word here is attempt. As noted by Levine (2012), "In many ways the idea of incorrect beliefs is fundamental to learning theory—if beliefs were always correct there would be nothing to learn about."

As individuals gain knowledge based on past choices and experiences in life, the "gap" between maximum possible utility (assuming this could be determined) and realized utility diminishes. As individuals come to realize the consequences of their choices on utility attainment they adjust their choices (behavior) when making subsequent choices. In this sense, information may be viewed as a flow and knowledge as a stock. The stock of knowledge is enhanced over time as the individual learns from past choices made. Whether the individual gains information by a Bayesian or non-Bayesian process is not relevant to the discussion here. The point is that the individual will attempt to maximize utility attainment by making choices based on his stock of knowledge which is gained from past choices and life experiences. We might refer to this process as strategic rationality. We are not concerned here with explaining the individual's internal cognitive processes, but simply to offer an explanation as to why expected utility and realized utility diverge. We simply assert that the individual learns (increases his stock of knowledge) from choices made and that this learning allows the individual to narrow the gap between utility he expects and the utility actually realized as the result of making choices. Realized utility converges towards maximum possible utility as choices are made and then evaluated by the individual. The concept that individuals have a learning curve is a useful tool for understanding and explaining why an individual's choices may not result in maximum possible utility attainment. We must note that this observation does not weaken the value of the traditional neoclassical assertion that individuals, when making choices, attempt to maximize utility attainment. Recognizing that decision makers have a learning curve merely provides an explanation as to why observed behavior (choice) can result in utility attainment that is less than maximum possible utility.

In the model below, the individual is viewed as if he goes through an iterative process as he attempts to make optimal choices over time.

## THE MODEL

Based on casual observation one likely would conclude (or surmise) that young children occasionally make choices that come nowhere close to maximizing utility. For example, they may overindulge in candy and ice cream to the point that it results in significant disutility. Similarly, adults also have been observed to make choices that almost certainly do not result in maximum utility attainment, e.g., drinking to the point of disutility. However, as children and adults gain knowledge their choices (as a group) will tend to improve. Therefore, one would expect a relatively large "gap" between maximum possible utility attainment and the utility actually realized as a result of choices made when individuals have little experience in making such choices. In an effort to explain this gap we assert that an individual goes through a learning process over time. At any point in time an individual's stock of knowledge is a function of past experience with choices made plus his education, both formal and informal. The individual attempts to maximize utility attainment over his time horizon and derives utility per unit of time. The individual's utility function is represented as, U[C(Xt,Kt(δ),t)], where (U) is utility, (C) is consumption (Xt) are choices made, K(δ)t is the stock of knowledge at some point in time, δ is a function representing efficiency with which knowledge reduces the gap between realized utility and maximum possible utility, and (t) represents time.

The function (U) determines the rate at which utility is gained at time (t) as the result of having K(δ)t stock of knowledge and making choices(Xt). The total utility that will be gained from some time (t) to some terminal date (T) is given by equation (1).

$$U[C(\dot{K}(\delta),t]dt, \tag{1}$$

where (C) is consumption, ($\dot{X}$) is the vector of choices over time, $\dot{K}$(δ)t represents the stock of knowledge over time, and (δ) represents the individual's efficiency in gaining and applying knowledge. At any point in time (t) the individual will have a certain stock of knowledge K(δ)t. With this stock of knowledge the individual makes choices ($\dot{X}$) which affect utility attainment. Then given his stock of knowledge at any particular time, together with current choices, the individual derives a certain rate of utility per unit of time. Choices made at any specific time are represented as X(t). Equation (1) then represents the sum of the rate at which utility is being gained in every small interval of time. The variables ($\dot{X}$) represent the time path of choice variables from an initial time (t=0) to the end of the individual's time horizon (T). If the individual follows a choice policy represented by ($\dot{X}$), he will gain total utility (U), which is the integral of utility gained in each small interval of time. We note that the individual, within limits, can choose the time path of choice variables ($\dot{X}$), however, he cannot choose independently, the stock of knowledge at each instant of time since his stock of knowledge depends on the stock at the initial date and time path of choice variables. This is because learning has occurred based upon choices made. This constraint can be stated by observing that the rate of change in the stock of knowledge at any instant of time is a function of the current stock K(δ)t, the date (t) and the choices made (X). We can represent this as equation (2),

$$\dot{K}(\delta) = dk/dt = \mathrm{f}(\mathrm{K}(\delta), \mathrm{X}, \mathrm{t} \tag{2}$$

so that choices made at any point in time will have two effects. First, choices influence the rate at which the stock of knowledge is changing. That is, the individual learns from choices made. So that, second, the stock of knowledge available for future choices has changed.

Equations (1) and (2) represent the problem of decision making over time. Essentially, the problem for the individual is to select the time path of choice variables ($\dot{X}$) in order to maximize utility attainment over time, while taking into account the effect of choices made on both the instantaneous rate of utility gained and the stock of knowledge carried into the future. This is a difficult problem, because the entire time path of choices has to be chosen. This difficulty can be overcome by reducing the problem to one which requires finding a few values (i.e., a single period problem). Following this procedure then, equation (1) can be generalized as equation (3).

$$U[C(K(\delta)t,\dot{X},t)]=\int_{O}^{T}U[\mathrm{C(K(}\delta\mathrm{),X,\,t)]dt} \quad (3)$$

which represents the utility that can be gained starting at an arbitrary time (t), with stock of knowledge K(δ)t and then following the choice policy , ($\dot{X}$) based on knowledge gained from previous choices till the end of the time horizon (T). Note that (U) can be separated into two parts. Consider a short time interval (Δ) beginning at some time (t), where we take (Δ) as being so short that the individual would not change his choices during the interval (Δ). This separation of utility is represented by equation (4).

$$U[C(K(\delta)t,X,t)]=U[C(K(\delta),X,t)]\,\Delta+\int_{t+\Delta}^{T}U[C(K(\delta),\mathrm{X,t)]dt} \quad (4)$$

Equation (4) indicates that if the stock of knowledge at time (t) is K $(\delta)_t$ and if the choice policy , ($\dot{X}$) is followed from time (t) forward, then the contribution to total utility from some time (t) forward is represented by the two terms in equation (4). The first term is the contribution in a short interval of time that begins at time (t). It is the rate at which utility is gained during the short interval multiplied by the length of the interval (Δ). So, the rate at which utility is gained depends on the current stock of knowledge K $(\delta)_t$ the current choices made (X) and the date (t). The second term in equation (4) is an integral beginning at time (t+ Δ). Note, the beginning stock of knowledge for the integral is not K (δ) but is $K_t$ (δ) t+Δ. Thus, the stock of knowledge will change during the interval (Δ). This is due to the influence on knowledge of the choices made (learning). The fact that knowledge changes (learning occurs) during the interval (Δ) is important. We can rewrite equation (4) as equation (5) in order to see this more clearly.

$$\mathrm{U[C(K(\delta)_t,\ \dot{X}\ ,\ t)] = U[C(K(\delta),\ X_t,\ t)].\Delta+U[C(K(\delta)_{t+\Delta},\ \dot{X}\ ,\ t+\Delta)]} \quad (5)$$

If the individual knew the best choice of ($\dot{X}$) from time (t) to the end of his time horizon (T), he could simply follow this choice policy ($\dot{X}$) and therefore maximize utility attainment over time ($\dot{U}$). We can represent this value of maximum utility by equation (6)

$$\mathrm{U^*\,[C(K(\delta),t)]=Max.\,U[C(K(\delta)_t,\dot{X}\,,t)]} \quad (6)$$

Notice that (U*) does not have ($\dot{X}$) as an argument. This is because ($\dot{X}$) has been maximized out. The maximum utility that can be had at time (t) with stock of knowledge $[(K(\delta)_t]$ does not depend on ($\dot{X}$), but is the value that can be had in those conditions from the best possible choice of ($\dot{X}$), the utility maximizing choices. Now, assume the choice policy ($\dot{X}$) is followed in the short time interval from (t) to (t+Δ) and from then on the best possible choice policy is followed. Referring to equation (6), the results of following this optimal choice policy can be represented as equation (7).

$$\mathrm{U[C(K(\delta)_t,X_t,t)]=U[C(K(\delta)_t,X_t,t)].\Delta+U^*\,[C(K(\delta)t+,t+\Delta)]} \quad (7)$$

The results of following the best possible choice policy are the benefits (utility) gained during the initial period (making choices $X_t$) plus the maximum possible utility that can be realized starting from time (t+ Δ), with stock of knowledge $K(\delta)_{t+\Delta}$ which results from choices made in the initial period. If the individual chooses the best value for ($X_t$), then (U) in equation (6) will equal

U*. Now differentiate $U[C(K(\delta)_t, X_t, t)]$ with respect to $X_t$ to get equation (8).

$$\Delta.\frac{\partial}{\partial Xt}U[C(K(\partial)_t X_t,t)]+\frac{\partial}{\partial Xt}.U^*[C(K(\partial)_{t+\Delta},t+\Delta)]=0 \quad (8)$$

However, the function (U*) in equation (8) is unknown but we need to differentiate (U*) with respect to (Xt), but (U*) does not involve (Xt) explicitly. However, we can write equation (9) as an approximation to equation (8):

$$\frac{\partial U^*}{\partial X_t}=\frac{\partial U^*}{\partial K(\delta)_{t+\Delta}}\bullet\frac{\partial K(\delta)(\delta)(t+\Delta)}{\partial X_t} \quad (9)$$

and since we are interested in short intervals of time, we can use the approximation $K(\delta)_{t+\Delta} = (K(\delta)t + \dot{K}(\delta)\Delta$. That is, the stock of knowledge at time (t+Δ) is equal to the stock of knowledge at time (t) plus the change in the stock of knowledge K(δ) (learning has occurred in the interval), multiplied by the length of the interval. Recall that $\dot{K}(\delta)=f(K(\delta),Xt,t)$ so we can write, $\frac{\partial K(\partial)t+\Delta}{\partial Xt}=\frac{\partial f}{\partial Xt}$ .

Note that $\frac{\partial U^*}{\partial K(\delta)}$ is the rate at which the flow of utility from time (t+Δ) to the end of the individual's time horizon (T) changes, with respect to the stock of knowledge available at time (t+Δ). We interpret this as the marginal value of knowledge at time (t+Δ). Let λ(t) represent the marginal value of knowledge at time (t) and be defined by equation (10).

$$\lambda(t)=\frac{\partial}{\partial K(\delta)}.U^*[C(K(\delta),t)]. \quad (10)$$

Now, substituting equation (10) into equation (8) we can write equation (11).

$$\Delta\cdot\frac{\partial}{\partial Xt}+\lambda(t+\Delta)\cdot\Delta\frac{\partial f}{\partial Xt}=0 \quad (11)$$

Canceling out the constant (Δ) since (Δ) is a very small value and asserting that the marginal value of knowledge changes over time, the approximate marginal value of knowledge at time (t+Δ) is the marginal value at time (t) plus the rate at which knowledge is changing during the interval multiplied by the length of the interval of time. This is represented by equation (12).

$$\lambda(t+\Delta)=\lambda(t)+\dot{\lambda}(t).\Delta \quad (12)$$

Now inserting equation (12) into equation (11) and canceling (Δ), we can write:

$$\frac{\partial U}{\partial Xt}+\lambda(t)\frac{\partial f}{\partial Xt}+\lambda(t)\cdot\Delta\frac{\partial f}{\partial t}=0$$

However, note that as (Δ) approaches zero (i.e. the time interval becomes smaller), the third term becomes very small. Then dropping the third term we can write equation (13).

$$\frac{\partial U}{\partial Xt}+\lambda(t)\frac{\partial f}{\partial Xt}=0 \quad (13)$$

Equation (13) indicates that along the optimal path for choice variables, at any time (t), the marginal short term effect of a change in choice must just offset the effect of that choice has on the total value of the stock of knowledge an instant later. The second term in equation (13) is the marginal effect of the current choice on the rate of growth of the stock of knowledge, where knowledge is valued at its marginal value (λ).

The individual will then attempt to make choices in each short interval of time so that the marginal immediate gain equals the marginal long term cost which is measured by the value of knowledge multiplied by the effect of choice on the accumulation of knowledge. Assuming that choices are made that satisfy equation (13) then, U[C(K(δ)t, X, t)] will equal U*[C(K(δ), t)]. Understanding that utility is affected by consumption (C) where choices ($\dot{X}$) made affect knowledge, we can now drop the C notation, and we can now write equation (14).

$$U^*[C(K(\delta),t)]=U[K(\delta),\dot{X},t].\Delta+U^*[K(\delta)_{t+\Delta},t+\Delta]. \tag{14}$$

Differentiating equation (14) with respect to the stock of knowledge $K(\delta)_t$ we obtain:

$$\lambda(t)=\Delta\cdot\frac{\partial U}{\partial K(\delta)}+\frac{\partial}{\partial K(\delta)}U^*[K(\delta)t,_{t+\Delta}]$$

$$=\Delta\cdot\frac{\partial U}{\partial K(\delta)}+\frac{\partial Kt+\Delta}{\partial K(\delta)}\cdot\lambda(t+\Delta)$$

$$=\Delta\frac{\partial U}{\partial K(\delta)}+\left[1+\Delta\frac{\partial f}{\partial K(\delta)}\right].(\lambda+\dot{\lambda}\Delta)$$

$$=\frac{\partial U}{\partial K(\delta)}+\lambda+\Delta\lambda+\lambda\frac{\partial f}{\partial K(\delta)}\ \Delta^2$$

Eliminating the last ($\Delta^2$) term, we may write equation (15)

$$\dot{\lambda}=\frac{\partial U}{\partial K(\delta)}+\frac{\partial f}{\partial K(\delta)} \tag{15}$$

where is the rate at which the stock of knowledge K(δ) is changing. Equation (15) indicates that when the optimal time path for accumulating knowledge is followed, the increase in the value of a unit of knowledge in a short interval of time is the sum of its contribution to utility in that time interval. Note, we can interpret $\dot{\lambda}$ as the decrease in utility that would be realized if the gaining of knowledge had not occurred.

In order to determine time paths of choice variables ($\dot{X}$), the stock of knowledge $K(\delta)_t$ and the value of knowledge (λ), we may write the augmented function equation (16).

$$H=U(K(\delta),X,t,+\lambda(t)\cdot f(K(\delta),X,t)]. \tag{16}$$

We can now determine partial derivatives and set them to zero.

If (H) is multiplied by (Δ) we can see that total utility gained in the time interval plus the gain in knowledge during the interval is valued at its marginal value. So that, (H·Δ) is the total contribution of choices that are made during the interval (Δ), including their direct contribution to the integral (U), and the value of knowledge accumulated during the interval (Δ). Therefore the choices made ($X_t$) during the current interval of time should be chosen to make (H) as large as possible. One way to accomplish this is to choose a value of (X) for which the partial derivatives vanish and we have done this. Rewriting equation (16) as equation (17)

$$H=U[K(\delta),X,t)+ d/dt \cdot \lambda K(\delta)], \text{ or } H = U[(K(\delta),X,t)+\lambda \dot{K}(\delta)+\dot{\lambda} K(\delta)] \qquad (17)$$

Where (H) represents the sum of utility gained in the interval (Δ) and the increase in the value of the stock of knowledge gained during the interval. Maximizing (H) with respect to choices (X) and knowledge K(δ) we get equation (18) and (19).

$$\frac{\partial U}{\partial X} + \lambda \frac{\partial f}{\partial K(\delta)} = 0 \qquad (18)$$

$$\frac{\partial U}{\partial K(\delta)} + \lambda \frac{\partial f}{\partial K(\delta)} + \lambda = 0 \qquad (19)$$

These equations tell the individual to choose the time paths of ($\dot{X}$) and ($\dot{\lambda}$) so that the resulting values of $K(\delta)_t$ are the ones he would choose if he could to make the sum of utility and the increment in the value of knowledge as large as possible in every short interval of time.

Consider now, based on the expressions derived above the following,

(a) $\dot{K}(\delta) = f(K(\delta), X, t)$

(b) $\frac{\partial U}{\partial X} + \lambda \frac{\partial f}{\partial X} + 0$

(c) $\frac{\partial U}{\partial K(\delta)} + \lambda \frac{\partial f}{\partial K(\delta)} = -\lambda$

The expression (a) specifies how knowledge changes in any small interval of time as a function of the current stock of knowledge and choices made (X). (Learning occurs)

Expressions (b) and (c) are the primary results of the maximization principle. Expression (b) indicates that the choices in every small interval of time should be made so that the marginal immediate gains are balanced with the value of the marginal contribution to the accumulation of knowledge. Expression (c) allows the value of knowledge to decrease as it is used.

These three expression (a), (b) and (c) jointly determine the optimal time path of choice for (X), the stock of knowledge $K(\delta)_t$ and the value of knowledge (λ).

Assuming the individual starts at time (t) with a given stock of knowledge and value of knowledge and writing expression (b) more exactly, we can write equation 20 as follows,

$$\partial/\partial X\, U(\dot{K}\,.\,(\delta)t, \dot{X}\cdot, t) + \dot{\lambda}(t)\cdot\partial/\partial X\, f(\dot{K}\cdot(\delta)t, \dot{X}\cdot, t) = 0 \quad (20)$$

where $\dot{K}(\delta)$ and $(\dot{\lambda})$ are known. This expression determines choices made (X). We can then substitute this value of (X) into expression (a) to drive $\dot{K}(\delta)_t$, the rate at which the stock of knowledge is changing. Then substituting this value of $\dot{K}(\delta)_t$, into expression (c) we get a value for $\dot{\lambda}$, the rate at which the value of knowledge is changing. Thus we can know the stock of knowledge and the value of knowledge for a short time interval into the future.

The point is, using these values we can repeatedly substitute into expressions (a),(b) and (c) to find a new value for the choice variables (X), a new rate for the change in the stock of knowledge $\dot{K}(\delta)t$ and new rate for the change in the value of knowledge $(\dot{\lambda})$. Continuing this substitution process again and again we can trace out the time paths of all variables from time (t) to the end of the individual's time horizon.

Again, it must be noted that we are not asserting that individuals actually solve this dynamic problem, but that through a process of learning from past choices, they will begin to behave (make choices) which will reduce the divergence between expected utility and realized utility. If individuals had perfect knowledge, then realized utility would equal maximum utility. However, individuals do not have perfect knowledge.

## CONCLUSION

Some argue that neoclassical utility theory is not supported by the facts (and their experiments) and that the idea of bounded rationality, or even irrationality, is a more "realistic" assumption than asserting that individuals behave rationally when making choices. They assert that human limitations prevent individuals from evaluating all possible choices available and thus thwart their effort to maximize utility. Those favoring bounded rationality or irrationality explanations for human decision making point to "real-world" experiments and examples that they assert violate rationality based utility theory.

On the other hand, utility theory based on a rational behavior assumption asserts that individuals behave as if they act rationally and that they attempt to maximize utility. However, since we cannot see another individual's utility function, it is not possible to know whether an individual actually achieves maximum utility. Nor can we know if firms attempt to act in their own best interests. The key benefit to this approach is that it yields clean predictions regarding choice. On the other hand, if we reject the rationality assertion, an endless number of possible human behaviors must be considered. We lose generality.

A primary drawback to asserting that individuals are less than rational is that few clearly testable predictions emerge from the many alternative behaviors that individuals might exhibit if they do not behave as if they are rational. As Becker, Glaeser and others have demonstrated, in the aggregate, predictions about how individuals respond in the marketplace to changes in environmental factors such as prices, incomes, taxes, etc. using the rationality assumption are supported by the actual choices individuals make.

We assert that individuals make choices based on their stock of knowledge—given the constraints or opportunities they face. We further assert that individuals are subject to a learning curve and that this phenomenon can explain why realized utility as the result of choices made may not equal the maximum possible utility attainable at the time (assuming we know maximum possible utility attainable). While actual utility attainable is not observable, the assertion that individuals attempt to maximize utility allows the prediction of behavior. Choices an individual makes are considered to be determined by the interaction of preferences (all the subjective things an individual would be likely to do or possess) and opportunities or constraints. Then, given preferences, constraints or opportunities are the determinants of choice. Opportunities are in principal observable and measurable, but preferences are neither. Since preferences are not observable, they must be asserted. For analysis purposes, it is asserted that whatever the individual's preferences are, they do not change erratically over the time period we're analyzing. So, given tastes, when opportunities change in an observable and measurable way, we expect choices made by individuals to change. Then these changes in choices made can be attributed to changes in constraints. It may not be possible to know the original choices made by individuals, but it is possible to predict how choices change when opportunities or constraints change. We seek to explain behavior based on changes in opportunities faced by individuals. Facts alone do not explain events. The stock of knowledge can explain the difference in the maximum utility and realized utility gained as the result of the individual's choice.

**ENDNOTES**

[1]Camerer and other behavioral economists refer to the need to "reunify" (as opposed to needing to form a brand new synthesis) economics with psychology, arguing that the divergence generally traces back to the efforts of theorists like Samuelson, Arrow and Debreu to formalize economics mathematically in the tradition of physics, plus the so-called "F twist" of Friedman that "allowed economists to ignore psychology." They argue that if one examines earlier works such as Adam Smith's Theory of Moral Sentiments (as opposed to focusing only on his Wealth of Nations), he/she will find the earlier analysis "shot through with psychological insights."

[2]The term "situationalism" is used here to represent the view that people isolate decisions and overweigh immediate aspects of the situation relative to longer term concerns. Consider for example, David Collander's (2006) endorsement of the argument that the economy must be analyzed as a "complex system" as opposed to a highly complex "simple system," meaning that "such complex systems are built up in path dependent stages, making individual optimization within such systems history- and institution-specific." Finally, he asserts, "This means that institutional structure is central to understanding a complex system."

[3]Though Roy Radner generally is credited with having introduced the concept of approximate Nash equilibrium or ε-equilibrium around 1980, it must be acknowledged that the idea of approximate optimization obviously is not new to economics. Clearly, it extends back at least to the work of Simon and his concept of "satisficing" in the 1950s. Interestingly, one of Radner's answers to the question of why one (oligopolistic) firm might be satisfied with a less-than-optimal response to the strategies of other firms had to do with the costs of discovering and using alternative strategies, i.e., the possibility that a truly optimal response might be more costly to discover and use than some alternative, nearly optimal strategy.

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