STATIONARITY, UNIT ROOTS, AND NETWORK EXTERNALITIES: AN MMORPG CASE STUDY

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ABSTRACT

This paper examines the role of network externalities during the product life cycle of the massively multiplayer online game *World of Warcraft*. Augmented Dickey-Fuller unit root tests were used to determine the stationarity of active subscriptions. Half-life and trend estimates suggest that information cascades played an important role in the network's early growth and game expansions conferred uncompensated direct benefits during the later stages of the product life cycle. These results suggest that viral marketing is most effective during a network product's growth phase, while traditional marketing becomes more significant over time. **JEL Classification**: D80, D83, D85, and D91

INTRODUCTION

Economic networks are systems of integrated interconnections sharing a common technical platform in which goods, services, and information flow between and among network members after first passing through a hub or switch. Networks have become a ubiquitous feature of the global economy.

Many features of traditional networks, such as airlines, railroads, and shipping companies that move large numbers of people, products, and parcels over long distances, also apply to virtual networks. Virtual networks are "linked" Internet connections comprising computers, servers, switches, software, and related technologies. The telecommunications industry, for example, uses the Internet and the World Wide Web to provide voice and data services. Virtual financial and commercial networks provide online access to retail shopping and auctions sites, over-the-counter equities, bonds, and foreign exchange markets, clearinghouse services, automated banking, and debit and credit cards, to name just a few. News and entertainment virtual networks integrate cable and television broadcasting, multimedia streaming, and electronic publishing. Online networks enable millions of "gamers" to interact in virtual role-playing environments. Social networks make it possible for individuals and groups to form online communities sharing similar backgrounds and interests.

NETWORK EXTERNALITIES

Much of the network literature describes the process whereby networks expand and contract, interconnections form, deform, dissolve, and reform. A distinguishing feature of markets for network goods is they generate positive feedback effects in which members receive uncompensated benefits as the network expands [see, for example, Easley and Kleinberg (2010, Chapters 16 and 17) and Economides (1996)]. If the increase in uncompensated benefits is substantial, network externalities may result in an upward-sloping demand curve. Network externalities may also help explain rapid increases in market demand during the introduction and growth phases of a network's product life cycle.

There are at least two complementary effects that generate uncompensated network benefits. *Direct effects* occur when users receive uncompensated benefits by aligning their decisions, actions, and behaviors following a product innovation or the introduction of a complementary technology [see, for example, Katz and Shapiro (1985), Arthur (1990), Economides (1996), Shapiro and Varian (1998), Easley and Kleinberg (2010, Chapter 17)]. An oft-cited example of this is the fax machine, which is of little or no value to a lone user, but which becomes exponentially more valuable as the number of integrated users with access to this technology increases. Similarly, online social networks confer exponentially increasing uncompensated direct benefits as social groups expand and subsume each other.[1]

Another externality contributing to a network's growth is *information effects* in which users make sequential decisions based on the observed behavior of others, even when their privately-held information suggests a different course of action. Information effects may culminate in an *information cascade* in which user behavior predicated on inference and innuendo feeds on itself.[2] The decision to abandon private information in favor of suppositions drawn from the observed behavior of others is frequently emotional and impulsive, which may account for spasmodic network expansions or contractions. Information cascades (also referred to as *herd behavior*) tend to be fragile since decisions based on incomplete or incorrect information are quickly reversed.[3]

Information cascades, such as speculative bubbles in financial markets, can be difficult to recognize as they occur, even when comprehensive, real-time, and high-frequency data is available. Even when information cascades are correctly identified, describing the transmission mechanism and the conditions that instigated the contagion can be elusive, especially when the underlying network architecture is not well understood. In spite of this, several important studies have contributed to an understanding of the dynamics of information cascades [see, for example, Lerman and Ghosh (2010), Alevy et al. (2007), Banerjee and Fudenberg (2004), Rogers (2003), Plott (2000), Bikhchandani and Sharma (2000), Allsopp and Hey (2000), Anderson and Holt (1997), and Banerjee (1992)]. While studies of virtual social and financial networks have resulted in a deeper understanding of information transmission mechanisms, the paucity of reliable and comprehensive data of conventional consumer network goods has handicapped the development of a more complete understanding of information contagions.

This study attempts to partially rectify this deficiency by analyzing the dynamic properties of active global subscriptions for history's most popular massively multiplayer online role-playing game (MMORPG)—*World of Warcraft* (WoW). The analysis begins with a brief review of the standard model of the market for network goods and its application to the product life cycle (PLC). The standard model suggests a theoretical framework for analyzing temporary, trend-reverting shocks associated with network effects. This review is followed by brief discussions of massively multiplayer online games (MMOG) and the emergence of WoW—the most successful massively multiplayer online role-playing game (MMORPG).

An impediment to the analysis of WoW network externalities is the absence of a comprehensive data set. The methodology used to resolve this data shortcoming is discussed in the data analysis section. This is followed by the application of augmented Dickey-Fuller (ADF) unit root tests to determine the stationarity of active subscriptions in each phase of the WoW PLC. Establishing stationarity is important since shocks become permanently embedded in nonstationary data resulting in never-ending exponential growth—a phenomenon that is not normally observed in the real world.

The analysis of stationary WoW active subscriptions is followed by a discussion of the data's dynamic properties. Estimated half-lives of temporary shocks can be used as proxies for the relative strengths of trend-reverting network effects during each phase of the PLC. These trend-reverting properties have important implications for network publishers' marketing strategies. The final section of this paper summarizes the main conclusions of this study and discusses the implications for viral and traditional marketing.

STANDARD MODEL OF THE MARKET FOR NETWORK GOODS

Network goods differ from pure private goods in that they exhibit positive feedback effects [see Katz and Shapiro (1985), Economides (1996), Shapiro and Varian (1998), and Easley and Kleinberg (2010, Chapter 17)]. According to the principle of diminishing marginal utility, the maximum price that buyers are prepared to pay for additional units of a pure private good decline with an increase in the quantity demanded.[4] In the case of network goods, however, an increase in network size bestows uncompensated benefits on incumbent members. The result can be an upward-sloping demand curve at low membership levels, such as during the introduction and growth phases of the PLC. The demand curve for network goods begins to assume its familiar downward-sloping shape as the network good matures.

The reservation price of consumer x in the standard model may be described by a compound inverse demand function of the form $f[E(x_t)]r(x_t)$, where x_t is the contemporaneous share of the population expected to join the network in period t, $r(x_t)$ is the compensated reservation price, and $f[E(x_t)]$ the consumers' uncompensated expected benefits.[5] The model assumes that single-unit users with perfect expectations are indexed in ascending order according to their reservation prices in the half-open interval $(0, 1]$.

To illustrate the structure of the standard model, suppose that reservation prices are linear and contemporaneous according to the equation $r(x_t) = 1 - x_t$, which has a parabolic shape. At a constant marginal cost, this model has two equilibria in the price interval $[0, ¼)$ [see Figure 17.3 in Easley and Kleinberg (2010, p. 455)]. For values of $p > ¼$, $x = 0$. A price increase in the interval $[0, ¼)$ results in an increase in the quantity demanded following a marginal increase in uncompensated benefits that exceeds the marginal decrease in compensated benefits. The result is a reversal of the law of demand.

The sources of these uncompensated benefits have been identified in the literature as the direct and information network effects discussed earlier. Network effects play an important role in defining the structure of this market. At low membership levels, the demand curve intersects marginal cost from below. Thus, equilibria for market shares $x_t < ½$, such would be found during the introduction and decline phases of the PLC, are unstable "tipping points," Temporary shocks in this region can be expected to accelerate network growth during an information cascade or hasten its demise. By contrast, stable market equilibria for market shares $x_t > ½$ when the demand curve is downward sloping are consistent with the maturity and decline phases of the PLC.

While shocks may occur during any phase of the PLC, incumbents and prospective users are less prone to make impulsive decisions once a network reaches maturity since by then the benefits of membership are well understood. For this reason, information cascades resulting from herding behavior are most likely to occur during the introduction and growth phases, which is consistent with the prediction of the standard model that tipping points exist at low levels of network membership. On the other hand, network shocks affecting network growth are more likely to be associated with the direct effects of product innovation or new technology during the maturity and decline phases.

MASSIVELY MULTIPLAYER ONLINE GAMES

The standard model of network goods assumes that single-unit users are indexed in ascending order according to their reservation prices in the half-open interval $(0, 1]$. The market for massively multiplayer online games (MMOGs) is a virtual network that satisfies this requirement. An MMOG is comprised of users (gamers), a World Wide Web protocol that formats and transmits gamer instructions (such as HTTP—hypertext transfer protocol), application servers that integrate gamers and servers, database servers that manage data storage and retrieval, and Internet service providers (ISPs). MMOGs simultaneously host millions of gamers in thousands of clusters in a continually updated interactive environment that accommodates a variety of Internet-capable platforms, such as personal computers, tablets, video game consoles, and smartphones. Access to online gameplay requires that individual users have dedicated active subscriptions.

Online games are major contributors to Internet traffic. Prior to the late-1990s, the development of graphic MMOGs was limited by capacity restrictions of dial-up modems. Beginning in the late-1990s, however, MMOGs experienced explosive growth due to the development of broadband Internet technology, which allowed for more complex graphics and audio features that enhanced the interactive gaming experience.[6] By 2015, the number of active global MMOG subscribers had grown to more than 1.5 billion gamers worldwide. This surge in MMOG's popularity was accompanied by an increasingly competitive online gaming industry in terms of the number of publishers, game genres, and titles.

There are several MMOG genres including first-person shooter (FPS) games, massively multiplayer online role-playing games (MMORPGs), racing games, sports games, social games, fighting games, and puzzle games. MMOG genres differ in terms of storylines, virtual environment, server updates, and speed of gameplay. MMORPGs and FPS games are the most popular MMOG genres in terms of active user subscriptions. FPS games are weapons-based combat scenarios experienced through the eyes of an avatar. FPS games are characterized by short sessions of rapid, drop in-and-out play.[7] By contrast, MMORPGs involve a large number of players interacting in virtual real-world, fantasy, science fiction, superhero, horror, and historical settings. Players create and develop a broad range of characters who complete a series of ever more challenging stages or "quests." MMORPGs are slower-paced than FPS games and involve more prolonged gameplay. Some MMORPGs even allow for an exchange of virtual currency.[8]

WORLD OF WARCRAFT

Prior to the release of *World of Warcraft*, the most popular MMORPG was *Lineage*, which was published in 1998 by South Korean video game developer NCsoft. By 2004, active *Lineage* global subscriptions exceeded 3 million gamers. In that same year, Blizzard Entertainment, Inc. (Blizzard) of Irvine, California released WoW.[9] In the next nine months, active WoW global subscriptions surpassed *Lineage*'s high-water mark. By mid-2008, *Lineage* active subscriptions had fallen below 1 million gamers, while active WoW subscriptions eclipsed 11 million (see Figure 1). Two years later, NCsoft had shut down *Lineage*, while WoW active subscriptions peaked at 12 million users.

The decline of the WoW franchise in the months that followed can be explained by a variety of reinforcing factors, including gamer ennui, evolving gamer tastes and preferences, and increased competition from rival online game publishers. After more than half a decade of MMOG market dominance, WoW had begun to show its age. Using the terminology of evolutionary biology, WoW became the victim of intragenus competition in which the dominant species became vulnerable to natural displacement by more successful subspecies.

DATA ANALYSIS

Identifying network externalities requires comprehensive, real-time, high-frequency temporal data in which stochastic disturbances are minimized. Much of our understanding of information transmission mechanisms and network architecture comes from empirical research of social networks and financial market transactions [see, for example, Lerman and Ghosh (2010), Hogg and Lerman (2009), Leskovec and Horvitz (2008), Alvey et al. (2007), Leskovec et al. (2007), Liben-Nowell and Kleinberg (2007), Leskovec et al. (2006), Vazquez et al. (2006), Gruhl and Liben-Nowell (2004), Wu et al. (2004), and Bikhchandani and Sunil (2000)]. The dearth of reliable and consistent high-frequency time-series data, however, has handicapped the development of a deeper understanding of the dynamics of such network goods as online games.

MMOG publishers tend to release comprehensive and consistent subscription data only when sales are robust, perhaps as a marketing ploy to stimulate product demand and burnish their corporate image. Blizzard, for example, routinely released detailed monthly data as active WoW subscriptions skyrocketed during the first 15 months following its debut in October 2004. As the sales growth slowed in early-2006, however, the release of subscription data became more erratic as Blizzard began reporting sales data in its quarterly earnings reports.

By the third quarter of 2015, active WoW subscriptions had fallen to around 5.5 million subscribers.[10] In September 2010, Blizzard announced that it would no longer release WoW subscription data, despite the fact that the total number of active subscriptions was still impressive by industry standards. The decision to suspend reporting sales data was widely interpreted as *de facto* recognition that WoW was nearing the end of its PLC.

In the 131 months following its debut, Blizzard released data on active WoW subscriptions on average every 1.6 months. To analyze the dynamic properties of active WoW global subscriptions a more comprehensive data set was needed. The preferred empirical method for approximating missing observations is to regress the available data against a highly-correlated proxy. Unfortunately, the search for a suitable proxy was unsuccessful. The less satisfying approach used in this study involved a two-step process. The first step involved linearly interpolating missing monthly subscription data. The resulting data set was then exponentially smoothed and the resulting estimates substituted for the missing data.[11] The data on active subscriptions used in this study are summarized in Figure 1.

AUGMENTED DICKEY-FULLER UNIT ROOT TEST

What is the evidence that the growth of WoW was at least partly attributable to the presence of network externalities? To answer this question it is necessary to determine whether active WoW subscriptions reverted to a long-run trend following temporary shocks, or did the data follow a random walk? If the data followed a random walk then we can conclude that network externalities played no role. On the other hand, a stationary time series suggests that direct and information network effects were not only present but had a persistent effect on future sales. This is an important consideration since it tells us something about the potency of word-of-mouth sales and the effectiveness of more traditional promotional efforts.

The test for stationary involves applying ordinary least squares (OLS) to estimate the parameters of an autoregressive time series given by the process

$$s_t = \alpha + \beta t + \rho s_{t-1} + u_t \tag{1}$$

where s_t represents active subscriptions at time *t*. If $\beta > 0$ and $\rho < 1$ then s_t is stationary after detrending. On the other hand, if $\alpha \neq 0$, $\beta = 0$ and $\rho = 1$ then s_t follows a random walk with "drift." This unmodified approach is problematic, however, since the Gauss-Markov conditions are violated. Standard tests of significance may not be valid because random walks do not have a finite variance. David Dickey and Wayne Fuller developed a test for determining the statistical significance of unit roots [see Fuller (1976) and Dickey and Fuller (1979, 1981)].

It is standard procedure when testing for random walks to include Δs_t in Equation (1) since s_t (even when detrended) can yield spurious results. Moreover, it is not possible to test whether the estimated value of ρ is statistically different from unity using a standard *t*-test. The reason for this is that when $\rho = 1$, OLS estimates are biased towards zero, which could lead to incorrectly rejecting the random walk hypothesis. Dickey and Fuller (1981) overcame this problem by deriving a distribution to test the hypothesis that $\beta = 0$ and $\rho = 1$. Sample critical F-values (F^*) are presented in Table 1.

An augmented Dickey-Fuller (ADF) unit root test proceeds as follows. First, assume an autoregressive process of the form

$$s_t = \alpha + \beta t + \rho s_{t-1} + \gamma \Delta s_{t-1} + u_t \qquad (2)$$

where $\Delta s_{t-1} = s_{t-1} - s_{t-2}$. Subtracting s_{t-1} from both sides of Equation (2) yields the *unrestricted* equation

$$\Delta s_t = \alpha + \beta t + (\rho - 1)s_{t-1} + \gamma \Delta s_{t-1}. \qquad (3)$$

After estimating Equation (3) with OLS, estimate the *restricted* equation

$$\Delta s_t = \alpha + \gamma \Delta s_{t-1}. \qquad (4)$$

To test the null hypothesis that $\beta = 0$ and $\rho = 1$, a Wald F-statistic is calculated using the error sum of squares (ESS) and degrees of freedom of the estimated unrestricted and restricted equations.[12, 13] Table 2 summarizes the OLS estimates of the unrestricted (U) and restricted (R) equations for the entire sample period and for each phase of the PLC.[14] Columns (2) to (5) summarize the parameter estimates for each regression. The numbers in parentheses are standard errors. Columns (6) and (7) report the corresponding ESS and degrees of freedom used to calculate the Wald F-statistics in Column (8).

Since $F_W > F^*$ at the 1 and 5 percent confidence levels, the random-walk hypothesis is rejected for the entire sample period and for each phase of the PLC. Since active subscriptions may be characterized as a stationary time series, knowing how long it takes for a temporary shock to revert to its long-run trend has important marketing implications because it tells us something about the persistence of direct and information external effects on network growth.

DYNAMIC PROCESSES

The analysis presented in the preceding section suggests contributed to the growth of the WoW network. This section examines the dynamic properties of this time series and attempts to identify information and direct network effects. To distinguish these network effects, the restricted equation used in the ADF unit root tests was modified to explicitly account for the presence of direct effects. What remains should include information effects, if any.

Recall that direct effects occur when network users receive uncompensated benefits by aligning their decisions, actions, and behaviors in response to product innovations or complementary technologies. A game expansion is an example of such an innovation.[15] During the period covered by this study, Blizzard released five expansions. The first of these gaming upgrades was *The Burning Crusade*, which was released in North America, Europe, Singapore, Thailand, and Malaysia on January 16, 2007 (indicated by 1 in Figure 1).[16] This was followed by its release in Australasia a day later; South Korea on February

1; Taiwan, Hong Kong, and Macau on April 30; and the Peoples Republic of China on September 30. This expansion was part of a marketing strategy to boost subscription sales. While the upsurge was substantial, sales growth continued to decelerate as WoW entered into the maturity phase of its PLC (See Figure 1).

Blizzard released two more expansions during the 3-4 years of the maturity phase. *Wrath of the Lich King* was released on November 13, 2008, followed by *Cataclysm* on December 7, 2010 (see ② and ③ in Figure 1).[17] While these expansions energized gamer interest, the effect on sales was less than for the first expansion. Blizzard responded to the downturn in new subscriptions with *Mists of Pandaria*, which was released on September 25, 2012 (see ④ in Figure 1).[18] By this time, WoW was well into the decline phase of the PLC. Although the fourth expansion increased sales by roughly 3 million subscriptions, the games downward sales trajectory resumed a month later.

Warlords of Draenor was released on November 13, 2014 (see ⑤ in Figure 2), more than two years following the release of *Mists of Pandaria*. Sales of this fifth expansion were a disappointing 2.7 million in the first week following its release. While impressive in its own right, this increase was the lowest of any previous expansion. To make matters worse, there was no apparent resurgence in gamer interest. By the start of the second quarter of 2015, active subscriptions had fallen to 7.1 million—300,000 fewer subscribers than before the release of *Warlords of Draenor*. On August 8, 2015, Blizzard announced that its global subscriber base had fallen to 5.6 million users—the lowest level since 2005. WoW was approaching its denouement.[19] By mid-2019, independent estimates put active WoW global subscriptions at around 4.5 million users.

To capture the direct effects of these expansions, Equation (3) was modified as

$$\Delta s_t = \alpha + \beta t + (\rho - 1)s_{t-1} + \gamma\Delta s_{t-1} + \delta d \qquad (5)$$

where $d_t = 1$ for the first and second month following the release of an expansion, and $d_t = 0$ otherwise. This dummy variable was set equal to unity for two sequential months to account for the benefits of an expansion to disseminate within the gaming community. Final parameter estimates and associated statistics for Equation (5) are summarized in Table 3.

Columns (2) and (3) of Table 3 summarize the estimated constants and time index parameters, respectively. The time index indicates whether active subscriptions exhibited a long-run trend. The parameter estimates in Column (4) are used to test for a random walk. The numbers in parenthesis are *t*-statistics to test the null hypothesis $\rho = 1$ (i.e., a unit root) against the alternative hypothesis $\rho < 1$. $(\rho - 1)$ not statistically different from zero implies that $\rho = 1$, i.e., a random walk. No random walk requires rejection of the null hypothesis that $(\rho - 1) = 0$ in favor of its alternative $(\rho - 1) < 0$.

Standard *t*-tests to determine statistical significance is inappropriate when s_t is nonstationary. Since the central limit theorem does not apply, $(\rho - 1)$ does not have the usual t-distribution. Once again, David Dickey and Wayne Fuller (1979, 1981) came to the rescue by calculating the asymptotic distribution of OLS estimates of $(\rho - 1)$ under the unit-root hypothesis. These critical values (DF_c) are reported in the square brackets below each *t*-statistic. If $t > DF_c$ (i.e., that $\rho = 1$) then it is not possible to reject the null hypothesis of a unit root, in which case we must conclude that active subscriptions follow a random walk, that is, there are no network effects. If $(\rho - 1) < 1$ then *t* will be negative. $DF_c < 0$ will lead to a rejection of the null hypothesis of a unit root.

Column (5) of Table 3 summarizes the estimated parameters of Δs_{t-1}. A statistically significant explanatory variable indicates that active subscriptions constitute a second-order autoregressive process. The addition of this variable was necessary to correct for serial correlation, which can inflate the estimated t-statistics and make t-tests unreliable. Column (6) tells us whether the release of new expansions had a statistically significant effect on new subscriptions. Columns (7) and (8) include test statistics for first-order serial correlation. Column (7) summarizes the familiar Durbin-Watson (DW) statistic. Column (8) reports Lagrange multiplier statistics where $LM = (n - 1)R^2$ follows a chi-square distribution. The numbers in parentheses are the associated critical values at the 5 percent confidence level $[LM_c = \chi^2(0.05)]$. We should reject the null hypothesis of first-order serial correlation when $LM < LM_c$.

Finally, Column (9) summarizes the estimated half-lives of temporary shocks, which were derived from the solutions to the corresponding first- and second-order difference equations. Half-lives indicate how long in months it will take for a temporary shock to decay by half. Estimated half-lives indicate the persistence of shocks to the network. For example, suppose that the release of a new expansion that initially boosts sales by 100 subscriptions has a half-life of two years. The number of sales accounted for by the expansion after two years is 50 thousand subscriptions; 25 thousand subscriptions two years after that, so on. An increase in a half-life translates into a greater overall impact on sales.

Recall that if $\beta > 0$ and $\rho < 1$, s_t will be stationary after detrending. The parameter estimates and statistics summarized in Table 3 support the findings of the ADF unit root tests in Table 2 that active subscriptions were stationary overall and for each phase of the PLC. That is, we reject the random walk hypothesis since $t > DF_c$. The results presented in Table 3 indicate that new expansions had a statistically significant effect on sales during the maturity and decline phases of the PLC. While new expansions boosted sales an average of 194 thousand active subscriptions overall, estimated half-lives steadily declined. New expansions during the maturity phase, which boosted sales by about 272 thousand active subscriptions, had a half-life of around 9 months. New expansions during the decline phase increased sales by about 1.5 million subscribers, although half-lives fell to less than two months.

Significantly, although the release of *The Burning Crusade* during the growth phase was statistically insignificant, temporary shocks had a half-life of almost 2 years. Moreover, there is no evidence of a positive trend during the growth phase, even though the WoW subscriber base expanded rapidly during this period. What accounted for the network's rapid growth? One possible explanation was word-of-mouth sales that led to an information contagion.

The above analysis supports the idea that active subscriptions were stationary; that network effects were trend-reverting, and that temporary shocks as measured by half-lives diminished over time. These results are amplified by examining the dynamic properties of the estimated equations in Table 3. The solutions to the corresponding first- and second-order linear difference equations are summarized in Table 4 and depicted in Figures 2 to 5.

Figure 2, for example, depicts the solution to the second-order difference equation for the entire PLC in Table 4. This solution assumes initial conditions of zero sales in period 0 $[s(0) = 0]$, sales in period $t = 1$ of 100 thousand subscriptions $[s(1) = 100]$, and no expansions $(d = 0)$. The dashed line illustrates the time trend during this period, while the solid line represents the time path of active subscriptions following a temporary shock for a period of 100 months (8.3 years). The reader can verify by inspection that the half-life of temporary disturbances during the entire PLC was

20.4 months. That is, it took about 1.7 years for a temporary shock to decay by half; another 1.7 years to decay by half again, and so on. Overall, it took about 7 years for a temporary shock to converge to within around 5 percent of the long-run trend.

SUMMARY AND CONCLUSIONS

This paper examined network externalities and their possible relationship to the product life cycle of history's most popular massively multiplayer online game—*World of Warcraft*. This study began with a discussion of two complementary network externalities. Network users receive direct benefits when they aligning their behavior following the introduction of innovative or complementary technologies. Information effects occur when network users make sequential decisions based on the observed behavior of incumbents.

This study found using augmented Dickey-Fuller unit roots tests that active global subscriptions during all phases of the product life cycle were stationary, i.e., that temporary shocks were trend reverting. The study also found that direct benefits to users from new game expansions were statistically insignificant during the introduction and growth phases of the product life cycle, but were important in boosting sales during the maturity and decline phases. This suggests that rapid early network growth may have been the result of word-of-mouth sales resulting in an information cascade. Finally, the persistence of temporary shocks that increased sales as measured by their half-lives was significant (almost 9 years) during the growth phase, but which steadily diminished over time. This suggests that efforts to boost sales by releasing new expansions met with declining success as *World of Warcraft* neared the end of its product life cycle, perhaps because of gamer ennui.

The practical significance of these results from a business perspective is the potential importance of viral marketing. Steve Jurvetson (2000) defines viral marketing as "network-enhanced word of mouth." According to Professor Jurvetson, "every consumer becomes an involuntary salesperson simply by using the product" (p. 110). This is a restatement of a network's information effect whereby current and prospective users make sequential decisions based on the observed behavior of others, even when privately-held information suggests following a different course of action.

Information cascades from viral marketing can be more effective than traditional advertising if it involves the implicit endorsement of trusted incumbent users. Ideally, word-of-mouth marketing will metastasize into an information cascade, such as occurred with the launch of email service provider *Hotmail* in 1996. *Hotmail*, which was acquired by Microsoft in 1997 for an estimated $400 million, included a clickable URL (web address) with each email to encourage recipients to adopt the service. The resulting network explosion resembled a viral contagion as *Hotmail*'s subscriber base grew from zero to 12 million users in 18 months—all on an advertising budget of just $50,000. Rather than waiting until its email service had seasoned, the early launch was an important contributing factor in the success of *Hotmail*'s marketing strategy. *Hotmail*'s success was not dissimilar to the experience of *World of Warcraft* during its introduction and growth phases.

ENDNOTES

1. Individuals may join social network websites or purchase fax machines because personal or business acquaintances have done so, or because the network provides access to a wider circle of potential contacts resulting in an increase in beneficial interactions. Lin and Lu (2011) argued that enjoyment is the most important factor in the decisions to join social networks, followed by the number of peers in the network, and the network's overall usefulness.

2. Information cascades are attributable to the work of Banerjee (1992), Welch (1992), Bikhchandani et al. (1992, 1998), and Milgram et al. (1969).

3. While these terms are frequently used interchangeably, Smith and Sørensen (2000) argued that information cascades occur when users ignore privately-held information when making sequential decisions, whereas this is not necessarily the case with herd behavior. The present study makes no effort to fine-tune these definitions.

4. Pure private goods are excludable in that payers can deny nonpayers from enjoying their benefits. In other words, private goods are rivalrous in that consumption by one user rules out the simultaneous consumption by others. Pure private goods do not produce third-party effects (externalities). Consumers and producers enjoy all of the benefits, but also incur all of the costs.

5. This compound function is a continuous and strictly monotonically increasing function of x_t. The inverse market demand equation $r(x_t)$ is assumed to be strictly decreasing and twice differentiable.

6. See Che and Ip (2012) and Chen et al. (2006) for an analysis of the impact of online gaming on Internet traffic. According to Che and Ip (2012), the growth of online gaming traffic volume poses serious challenges to servers and ISPs that rely on the efficient flow of Internet traffic.

7. Non-MMORPG genres, such as FPS *Counter Strike* (released in 1999), are relatively parsimonious in terms of their data and system requirements. User instructions comprise a few simple commands, such as "walk," "chat," "rest," and "attack." By contrast, narratively elaborate MMORPGs place much greater demands on the Internet infrastructure. According to Chen et al. (2006), the number of active MMORPG subscriptions exceeded 500 million in 2005, with the six most popular titles accounting for 3-4 percent of total Internet traffic.

8. Virtual currency issued by software developers circulates as a medium of exchange among the members of online gaming communities. Gamers use virtual money to purchase "add-ons," such as maps, extended storylines, antagonists, weapons to enhance the gaming experience. "Gold farmers" sell accumulated virtual currency for cash on websites that are separate from the game itself.

9. Blizzard Entertainment is a subsidiary of Activision Blizzard. Founded in 1991 as Silicon and Synapse, the company changed its name to Chaos Studios in 1994.

In 2008, Activision Publishing merged with Vivendi Games. In 2013, Activision Blizzard was born following a takeover of its then majority shareholder Vivendi.

10. According to *Guinness World Records*, *World of Warcraft* is the most popular MMORPG in history. In January 2014, Blizzard announced that more than 100 million user accounts had been created over the game's lifetime.

11. The exponential smoothing formula used in this study was $s_t = \alpha x_t + (1 - \alpha)s_{t-1}$, where s_t is the smoothed observation, x_t is the current observation, and $0 < \alpha < 1$ is a smoothing factor. The values of the smoothing factor that are close to unity have a lesser smoothing effect and give greater weight to recent observations. Values of α closer to zero have a greater smoothing effect and are less responsive to recent changes. Trial-and-error selection of the smoothing factor reflects the author's judgment, which in this study was $\alpha = 0.3$.

12. The test will remain the same when additional Δs_t lags are added to the right side of Equation (2).

13. The Wald *F*-statistic was calculated as $F_w = [(ESS_R - ESS_U)/(df_R - df_U)]/(ESS_U/df_U)$.

14. Phases of the PLC were subjectively identified and include one-month overlaps.

15. An online game expansion supplements an existing MMOG. Expansions extend existing storylines, introduce new quests, stages, avatars, virtual environments, combat scenarios, weapons, appliances, utilities, and so on.

16. Although the launch of *The Burning Crusade* was plagued by server crashes, sales of nearly 2.4 million in the first 24 hours made WoW the fastest-selling online computer game up to that time.

17. On its first day of release, *Wrath of Lich King* sales were more than 2.8 million. On its first day, *Cataclysm* sales were more than 3.3 million. Both releases solidified WoW's reputation as the most successful online computer game ever.

18. Sales of *Mists of Pandaria* were a disappointing 2.7 million in the first week following its release.

19. Blizzard released a sixth WoW expansion (*Legion*) on August 30 2016, and on November 3, 2017, released a seventh WoW expansion (*Battle of Azeroth*). Both of these releases fall outside the period examined in this study. An eighth expansion, *Shadowlands*, was announced by Blizzard, on November 1, 2019

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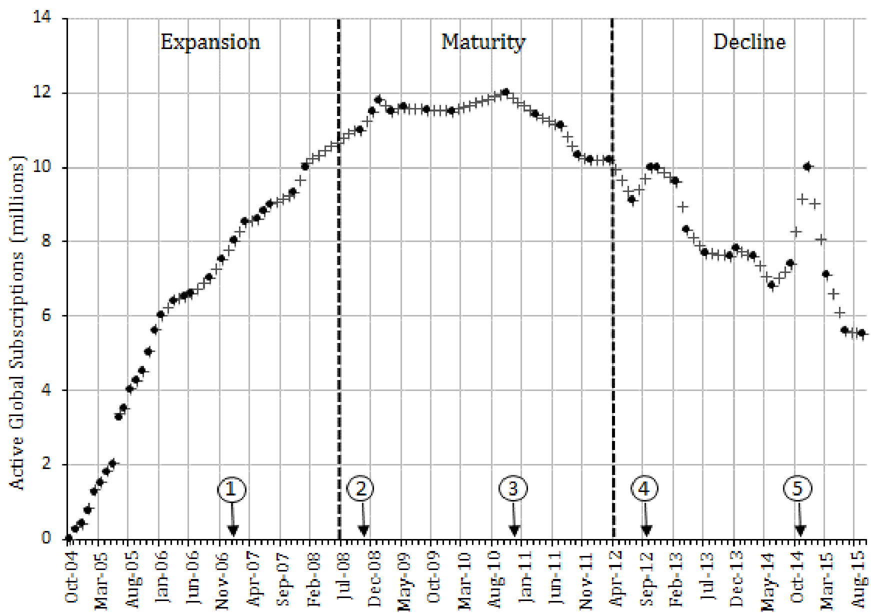
FIGURE 1
***WORLD OF WARCRAFT* PRODUCT LIFE CYCLE**



Notes: Actual (•) and fitted (+) subscription data.

TABLE 1
CRITICAL DICKEY-FULLER *F DISTRIBUTION**

	Probability of a smaller value							
N	0.01	0.025	0.05	0.10	0.90	0.95	0.975	0.99
25	0.74	0.90	1.08	1.33	5.94	7.24	8.65	10.61
50	0.76	0.93	1.11	1.37	5.61	6.73	7.81	9.31
100	0.76	0.94	1.12	1.38	5.47	6.49	7.44	8.73
250	0.76	0.94	1.13	1.39	5.39	6.34	7.25	8.43
500	0.76	0.94	1.13	1.39	5.36	6.30	7.20	8.34
∞	0.77	0.94	1.13	1.39	5.34	6.25	7.6	8.27
se	0.004	0.004	0.003	0.004	0.015	0.020	0.032	0.058

Source: Dickey & Fuller (1981), Table VI, p. 1063.

TABLE 2
DICKEY-FULLER TESTS *WORLD OF WARCRAFT* PRODUCT LIFE CYCLE

(1) Phase	(2) α	(3) β	(4) $(\rho-1)$	(5) γ	(6) ESS	(7) df	(8) F_w
PLC (U)	404.33	−2.65	−0.02	0.26	1.16E+07	123	61.97
	(105.09)	(0.85)	(0.01)	(0.09)			
PLC (R)	22.04			0.42	1.34E+07	126	
	(29.01)			(0.08)			
Growth (U)	503.93	17.07	−0.10	0.03	1.69E+06	34	38.50
	(116.03)	(14.36)	(0.06)	(0.17)			
Growth (R)	226.31			0.13	2.07E+06	36	
	(57.69)			(0.17)			
Maturity (U)	1334.26	−5.35	−0.09	0.22	2.35E+06	55	49.19
	(491.23)	(1.83)	(0.04)	(0.13)			
Maturity (R)	−6.88			0.29	2.77E+06	57	
	(28.69)			(0.13)			
Decline (U)	3248.54	−14.30	−0.22	0.44	6.29E+06	29	28.37
	(1730.62)	(10.56)	(0.09)	(0.17)			
Decline (R)	−92.02			0.32	7.52E+06	31	
	(88.75)			(0.17)			

Note: Numbers in parentheses are standard errors.

TABLE 3
EQ. (5) FINAL PARAMETER ESTIMATES.*

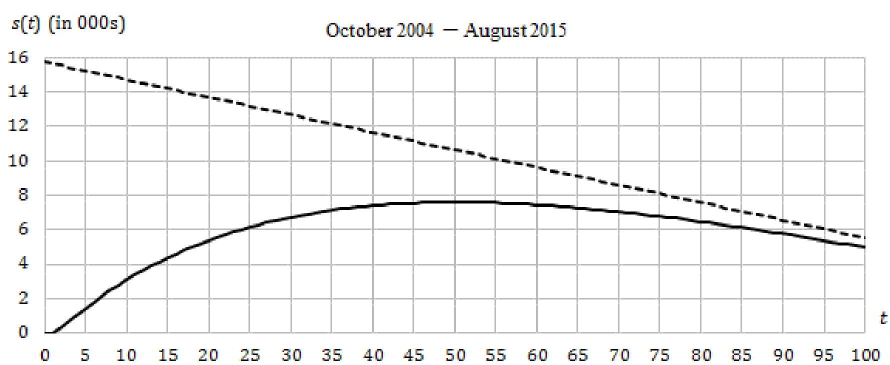
(1) Phase	(2) α	(3) β	(4) $(\rho - 1)$*	(5) γ	(6) δ	(7) *DW*	(8) *LM***	(9) *Half-life*
PLC	408.83 (4.14)	−2.65 (−3.20)	−0.03 (−2.48) [−3.45]†	0.217 (2.53)	193.96 (2.26)	1.96	0.45 (165.85)	20.4
Growth	447.38 (5.45)		−0.03 (−2.53)† [−2.94]			1.98	0.09 (59.91)	22.0
Maturity	1247.80 (2.69)	−6.27 (−3.75)	−0.08 (−2.13)† [−3.49]		272.28 (3.23)	1.64	1.55 (84.75)	8.7
Decline	5853.05 (3.51)	−30.43 (−3.02)	−0.33 (−3.88)‡ [−4.26]		1504.89 (4.40)	1.70	0.97 (52.21)	1.7

* Numbers in parentheses are t-statistics. Numbers in square brackets are right critical Dickey-Fuller values.
** The critical Lagrange multiplier values for the test of serial correlation are in the parentheses. Accept the hypothesis of no serial correlation when.
† It is not possible to reject the null hypothesis of a unit root ($\rho = 1$) at the right 0.05 critical value.
‡ It is not possible to reject the null hypothesis of a unit root ($\rho = 1$) at the right 0.01 critical value

TABLE 4
ACTIVE SUBSCRIPTION DIFFERENCE EQUATIONS

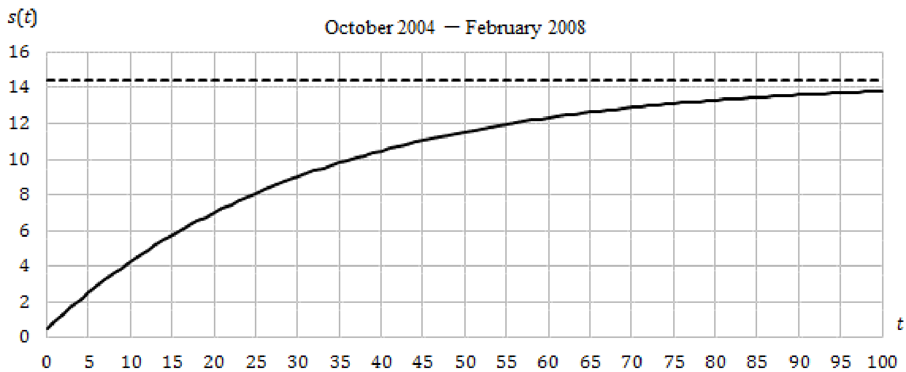
Phase	Solutions	Figure
PLC	$s_t = -16{,}286.28(0.97)^t + 561.97(0.23)^t + 15{,}724.31 - 101.92t$	2
Growth	$s_t = 500(0.97)^t + 14{,}431.61$	3
Maturity	$s_t = -15{,}705.16(0.92)^t + 16{,}205.16 - 81.36t$	4
Decline	$s_t = -17{,}024.11(0.67)^t + 1{,}752.11 - 91.11t$	5

FIGURE 2
SOLVED PLC SECOND-ORDER DIFFERENCE EQUATION



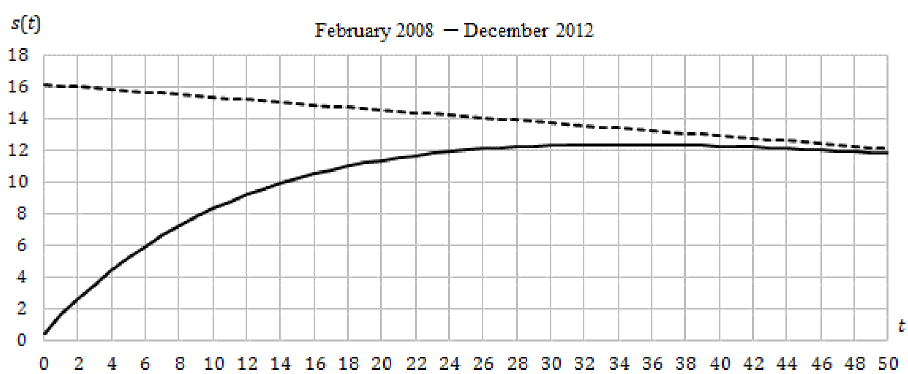
Note: The assumed initial conditions are $s(0) = 0$; $s(1) = 100$, $t(0) = 0$, $t(1) = 1$, and $d = 0$.

FIGURE 3
SOLVED GROWTH PHASE FIRST-ORDER DIFFERENCE EQUATION



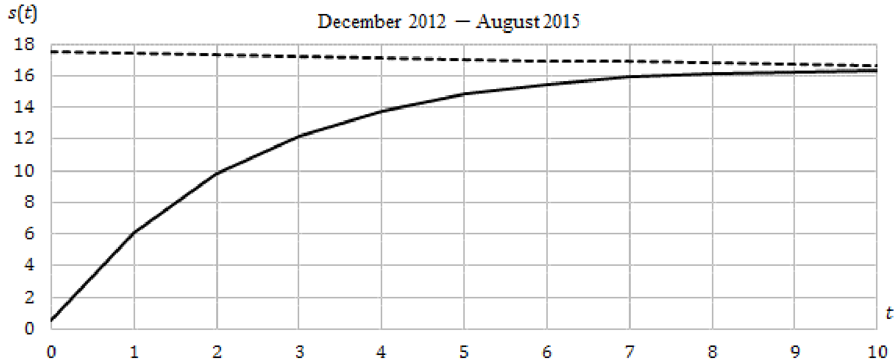
Note: The assumed initial conditions are $s(0) = 500$ and $t(0) = 0$.

FIGURE 4
SOLVED MATURITY PHASE FIRST-ORDER DIFFERENCE EQUATION



Note: The assumed initial conditions are $s(0) = 500$ and $t(0) = 0$.

FIGURE 5
SOLVED DECLINE PHASE FIRST-ORDER DIFFERENCE EQUATION



Note: The assumed initial conditions are $s(0) = 500$, $t(0) = 0$ and $d = 0$.