UNDERWATER AND UNDERSTUDIED: *UNTERSEEBOOTEN* IN THE FIRST WORLD WAR

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ABSTRACT

This paper applies production theory to the German U-boat campaign in World War One. A time-series dataset of the number of merchant ships sunk by U-boats and the extent of the U-boat capacity to sink allied ships is constructed from multiple sources, and connections are made between the usage of U-boats and the political concerns of the German government about drawing the U.S. into the conflict. A Cobb-Douglas functional form is used to estimate the effect of additional U-boat usage on ships sunk. The results show a strong and significant marginal product of additional U-boats: adding an extra U-boat patrolling for the typical amount of time a U-boat would spend at sea results in 10 more ships sunk per month while the corresponding estimate for losses in shipping tonnage indicates an additional 22,482 tons sunk. The length of time an average U-boat spent on patrol has no effect on either ship losses or tonnage sunk. Based on this analysis, Germany's ability to successfully blockade England though interference with merchant shipping would have been substantially improved by building and launching more U-boats. **JEL Classification:** N44

INTRODUCTION

Economist infrequently study war as a microeconomic, production problem. The Great War is no exception. Not to say economists have been mute on the subject of the relationship between the state of a country's resources and its ability to wage war, but the analysis has been conducted from a macroeconomic perspective, an example of which is Broadberry and Harrison (2005). After noting the dearth of quantitative economic analysis about World War One, these authors show the level of economic development of a country largely determines the degree of resource mobilization and, thus, the outcome of the war. The conclusion is based on measuring macroeconomic resources such as populations, territories, and gross domestic product of the combatants. Ferguson's (1999) work is somewhat closer to the idea of war as production. The author's approach takes total expenditures on the military in the various combatant countries and their military deaths and computes expenditures per death ratios. The author concludes that Germany was, by this measure, highly efficient. While Ferguson's results successfully widen understanding of the First World War, the scope continues to be macroeconomic in nature.

Production theory analysis encourages one to think about the organization of inputs and the resulting outputs. For a land war, the inputs are fairly clear: labor input in the form of soldiers, stores, generals, and capital comprising guns, artillery, tanks, airplanes, and the like. The appropriate measure of output is debatable (Ferguson, 1999) but ultimately rests on some measure of total enemy causalities. The estimation of the production function for a land war would prove challenging, owing to the disorganized nature of the battlefield. The application of microeconomic production theory to Germany's U-boat war promises a more successful application. At sea, the battlefield is less disorganized, and the inputs and outputs are more clearly identified. No example of production analysis applied to the U-boat war can be found in the literature. The estimation of the productivity of German U-boats in sinking allied merchant shipping thus forms the contribution to the literature of this endeavor.

The organization of the remainder of the paper conforms to the following route. Section Two describes the characteristics of the U-boat war and the integration of the characteristics into a production function framework. Section Three describes how the dataset is assembled. The fourth section considers trends in the effectiveness of U-boats, particularly with respect to the impact of certain political decisions made by German government on the military use of U-boats. The fifth section discusses the development and estimation of an empirical production function; the sixth reviews the estimation results. The final section offers conclusions and suggests future avenues of research.

DESTRUCTION OF MERCHANT SHIPPING IN A PRODUCTION FRAMEWORK

Of all three modes of 20th century warfare—land, sea, and air—Germany's U-boat campaign is best suited for cleanly identifying the outputs and the inputs. The objective of deploying U-boats is to disrupt the enemy's ability to sustain itself through trade by interfering with merchant shipping and to impeded the enemy's ability to conduct offensive actions at sea by sinking warships. The output is apparent: sunk ships. The size of the ship sunk is also a component of the output. The larger the ship, the greater the tonnage lost and hence the greater the disruption to the enemy's operations at sea. The inputs are also straight forward: the number of U-boats and the intensity of their use. To sink a ship, the U-boat had to be at sea. The foremost input variable to speak of is the density of U-boats at sea, measured in two different and complementarity ways. The first measure of U-boat density of to enter into the production function is the number of U-boats patrolling the waters. Several different measures of the quantity of U-boats at sea present themselves. The merits of the alternative measures will appear later. Whatever the final choice, variation in the number of U-boats should be termed variation at the extensive margin. Additionally, the length of each U-boat's patrol should be a productive input in sinking ships. For a fixed number of U-boats, an increase in the number of days each U-boat spends at sea should increase the amount of ships sunk. The length of U-boat patrols is a measure of the intensiveness of usage and fluctuation here will be termed variation at the intensive margin. Again, alternate measures of duration exist and will be sorted out later.

Beyond the extensive and intensive use of U-boats, the type of U-boat needs inclusion in the production function to capture any technological effect. During World War One, Germany used three types of U-boats. From the outset of the war, German had oceangoing, attack boats; the so called “U” type of boat. These were relatively large boats with crews ranging in size from 35 to 62 men and a torpedo complement ranging from 6 to 24 torpedoes. Either an 88mm or 105mm deck gun was present. These were designed to have lengthy patrols in large bodies of water like the North Sea.

During 1915, two new U-boat types were introduced. The UB and UC class boats were both designed for costal operations exclusively. The UB class was the attack platform, i.e., they were equipped with torpedoes. It was maned by between 14 to 34 men and equipped with between 2 and 10 torpedoes. Most were equipped with an 88mm deck gun. The UC class entered service shortly after the UB’s and were designed primarily as minelayers. The first version of the UC’s had a crew of 14 men and carried 12 mines. In the second and third variants, seven torpedoes were added and either 18 or 14 mines were carried. Most of the UC’s were also fitted out with an 88mm deck gun. Regardless of either the environment it was designed for or the nature of its weaponry, the goal of all three types of U-boats remains the same: sink ships. The way in which these three types of U-boats go about sinking ships is, however, varied and measurement of differences in effectiveness would be of interest.

The weapon of choice should likewise impact the amount of ships sunk. U-boats can sink ships in one of four ways: 1) placing a bomb on board to scuttle the ship; 2) using gun fire from the deck gun; 3) firing a torpedo; 4) if so equipped, by setting a mine. The U-boat’s uniqueness as a weapon comes from its stealth. A U-boat can approach a target without the target necessarily realizing it is being stalked. Enhancing the stealth feature, a U-boat can sink ships while maintaining its stealth either directly via a torpedo or indirectly with a mine. World War One torpedoes had a maximum range of 8,400 meters; more commonly 5,000 meters which is more than enough to maintain its furtiveness. Alternatively, a mine can be planted underwater, the U-boat can scurry away, leaving the placement of the mine unknown, again ensuring the continued stealth of the U-boat. The effect of the deck gun would obviously be enhanced when the U-boat surfaces from its position of stealth to the surprise of the merchant ship or warship. This method of sinking is quite, though not perfectly, similar to the one used by warships and not fully unique to the U-boat. Besides, when the U-boat surfaces it becomes much more vulnerable than a warship. The same argument applies to setting a bomb. A warship can certainly stop a ship and set a bomb and do it at a considerably smaller risk than a U-boat. The point here is that when describing the inputs to be included in a production function of sinking tonnage by U-boats, weapons unique to the U-boat are of the greatest interest.

The final factor worthy of inclusion is whether warning was given before the sinking. Clearly, sinking without warning, when combined with a torpedo, gives the U-boat total stealth. Total stealth should produce greater losses of ships because the ships cannot take defensive action, i.e., fire a deck gun at the U-boat and the U-boat can maneuver closer to the ships. It would also increase the number of causalities as they would have little time to man the lifeboats. The giving of a warning needs then to be accounted for in the production function. But including the giving of a warning as an explicit factor is also necessary as it reflects changes in governmental policy. The question of providing warning before sinking was as much, if not more, a political issue as a military one. Controlling for changes in Germany's political policy towards warning is necessary to ensure ceteris is indeed paribus.

The follow equation is a proposed specification of the production function:

$$Tons_t \text{ or } Ships_t = f(U\text{-}boats_t, Duration_t, Type_t, Weapon_t, Warning_t, \varepsilon_t) \quad t = 1,\ldots,T \tag{1}$$

The amount of tonnage lost or number of ships sunk is a function of *U-boats*, a measure of the number of U-boats at sea; *Duration*, a measure of the length of U-boat patrols; *Type*, a control variable for the type of U-boat; *Weapon*, a control of which weapon is selected for use; *Warning*, a control for the type of contact between the ship and U-boat prior to sinking; and ε, a random error term.

DATASET DEVELOPMENT

Investigation and estimation of the proposed production function requires data on the ship's tonnage, some relevant characteristics of the ships, e.g., is it a merchant ship or a warship, and measures of U-boat activity. Unfortunately, no such dataset exists; one had to be constructed. The data requirements fall into two categories. First, information was collected on the ships that were sunk by U-boats during the Great War. Second, information on the activities of U-boat fleet was accumulated.

Construction and Description of Microdata on Ships Sunk

Collection of the data on the sunk ships had two phases. The first phase involved pulling together different sources of information to create a microdata file containing all of the ships sunk during the Great War. Each separate sinking, ordered by the date of occurrence, formed an observation. Note that multiple sinkings frequently happened on a single date, so each one became an individual row of data. On other days, zero sinkings occurred. Four different sources make up the basis of the microdata set. The primary data source was *Uboat.net* (http://www.uboat.net/wwi/) a site devoted to collecting and presenting data on the German U-boat campaigns in both world wars. Two other websites were consulted: *Wrecksite.eu* (http://www.wrecksite.eu/wrecksite.aspx) and *Miramar.org* (http://www.miramarshipindex.org.nz/). *Wrecksite.eu* asserts it is the world's largest online wreck database. *Miramar.org* claims to be the most comprehensive index of ships and their histories, covering iron and steel sailing ships, powered merchant ships (over 100 tons) and warships, since the early 19th century. Both sites dispense information on all shipwrecks, not just ones due to World War One U-boats. *Wrecksite.eu* had a reference to every sinking listed on *Uboat.net* but sometimes their entry contained little information. *Miramar.org* did not have entries for some of the sinkings listed by *Uboat.net*. The 100 GRT cutoff point disallows the smaller ships. The two sites were mostly used to collaborate the information collected at *Uboat*.net. The microdata set collection began with the first sinking, the HMS

Pathfinder, on September 5th, 1914, and ended with the *Modiva* on January 31st, 1917.

Not all entries in the *Uboat.net* list were included. Specifically, only ships that were actually sunk were included. Any sinking caused by an Austrian, as opposed to a German, U-boat were also excluded. The justification is simple and straightforward: the Austrian U-boat campaign never amounted to much. The Austrian navy, over the course of the entire conflict, only possessed 26 U-boats, succeeding in sinking a paltry 124 ships of the 5,000 ships sunk in total. Next, when a specific U-boat could not be uniquely associated with a sunk ship, the observation was excluded. This exception was triggered when either no U-boat was associated a sunk ship or one of several possible U-boats caused the sinking. This exception applied in only a very few cases. Of the 2509 incidents between U-boats and ships occurring between August, 1914 and January, 1917, applying these restrictions reduced the total to 2,231 observations, which in turn meant there were 2,231 ships sunk (or "events") available for analysis.

Table 1 lists the variables with numerical content associated with sunk ship microdata. They are discussed in turn and any additional assumptions or restrictions applied will be noted. *TONS* is measured in gross register tonnage (GRT), defined as the volume of space within the hull and enclosed space above the deck of a merchant ship which are available for cargo, stores, fuel, passengers and crew. Of all the variables collected, GRT is by far the one least likely to show variance between sources presumably owning to its use for determining harbor dues and fees. From August, 1914 to January, 1917, the smallest ship sunk had a GRT of 5, the largest was 48,158 with a mean value of 1800 GRT.

NATIONALITY is the country whose flag the ship was sailing under. During the course of the entire war, 28 different countries had at least one ship sunk by a German U-boat. During the period of the current dataset, a subset of 21 countries lost ships. Many of those countries, 9 specifically, had fewer than 10 ships sunk. The top five countries experiencing events accounted for 87 percent of the ships lost. Britain had the most at 1,110 (50 percent); Norway was second with 286 (13 percent); France was third with 257 (12 percent); Italy, fourth at 199 (9 percent); fifth, Denmark with 81 (4 percent). That Britain lost the most ships by an overwhelming amount comes as no surprise. From the standpoint of data accuracy, it actually proves helpful because for British sinkings, as many as five different sources can be consulted to verify the information being collected.

LOCATION measures the area in which the ship was sunk, separated into six areas: the North Sea; the English Channel; the Atlantic Ocean; the Baltic Sea; the Mediterranean Sea; and all other locations. The most frequent location of a sinking was the North Sea at 31 percent of the sample. The Atlantic was second with 28 percent, and the Mediterranean was third at 23 percent.

The next two variables describe the purpose of the lost ship. The variable *WARSHIP* equals 1 for a warship, while *MERCHANT SHIP* equals 1 for a merchant ship. Firstly, the lion's share of the events involves merchant ships. A total of 2,040 merchant ships were sunk or 91 percent of the total; warships losses totaled 191 ships. Within the two categories, further distinctions emerge. For warships, the most likely type of ship to sink was a trawler (44 percent); next most common were minesweepers/minelayers (15 percent). The most common merchant ship sunk were cargo steamers (57 percent) followed by sailing vessels (22 percent). *CARGO*, applicable only to merchant ships, is a categorical variable for the type of cargo carried by the merchant ship. The most frequently occurring type of cargo was coal at 25 percent; timber claimed second place with 17 percent.

Of the 2,231 sinkings, 465 of them involved *CASUALTIES* which works out to 21 percent, with a mean of 39 people. Furthermore, of the 465 events with casualties, 304 involved merchant ships. For merchant sinkings with loss of life, the mean was smaller at 17 casualties with the maximum, 1,198, being the *Lusitania*. For warships, note that nearly all warship sinkings had casualties: 161 of the 191 with an average number of casualties of 80. A strong assumption was used here; which likely biases the estimated average casualty counts toward zero. Most of the entries at *Uboat.net* had no information on number of casualties. In some cases, *Wrecksite.eu* or *Miramar.org* or both provided information on casualties and this number was used. For sinkings lacking data at all three sites, the number of casualties was set to zero. The justification for the assumption rests with the observation that in most cases when one of the three sources did provide data, the number was zero. Certainly, if significant loss of life occurred, the Allies had strong incentives to report the figures. But in the cases of small loss of life, it is less clear that much attention would be paid.

ARMED MERCHANT SHIP is a dummy variable indicating if the ship was armed with an ability to either actively sink enemy ships or to defend itself from attack. All warships were assumed to be armed. As stated earlier, British data on their merchant ships sunk was the most complete so the number of defensively armed, British merchant ships should be the most accurate. For other countries, it is likely the number is understated. Overall, 97 merchant ships, representing 4 percent of the total, are defensively armed. The "Q Ships", disguised as unarmed merchant ships but possessing offensive purpose are counted as warships rather than merchant ships.

NO WARNING is an indicator variable for the way in which the ship was attacked. All ships sunk were either sunk without warning (1) or captured (0). All warships were sunk without warning. For merchant ship, they could be sunk without warning or "captured." The means by which a merchant ship could be captured i.e., sunk with some sort of warning, were multiple. The most common was for the crew to be given a certain amount of time to disembark and then the ship was sunk. Of the 2,040 merchant ship sunk, 435 of them or 21 percent had no warning. As with casualties, a strong assumption was used here: unless some specific information was found to indicate a sinking was without warning, then it was assumed to be captured.

The last variable is *WEAPON* of choice. The variable indicates how the sinking was accomplished. Five categories were defined: bomb; gunfire; torpedo; mine; unknown. The first three methods accounted for 75 percent of the sinkings and with approximately equal shares. For merchant and warships combined, bombs sank 27 percent, gunfire at 24 percent and torpedo at 23 percent. For warships alone, mines took 63 percent to the bottom while torpedoes sank 30 percent. For merchant ships, bomb was the most common way with 30 percent followed by gunfire, 26 percent, and torpedo, 22 percent. Unfortunately, "unknown" is a frequent outcome for merchant ships at 12 percent. This reflects a decision to attribute a method to a sinking only if one or more of the sources had specific information. In some cases, if a given U-boat has sunk 3 ships by gunfire, then one by unknown means, and then 2 more by gunfire, it might be reasonable to assume the unknown observation was gunfire. However, too often went complete information was available on a given U-boat, variation of method was noticeable, i.e., 3 gunfire, 1 torpedo, 2 gunfire. Given the importance of method of sinking to future analysis, the most conservative rule was used. Additionally, any time a mine caused a sinking, the assumption was made to set *WARNING* to "no warning."

Transition from Ships Sunk Microdata to Ships Sunk Monthly Data

The statistical analysis of the microdata reveals some interesting insights into the pre-1917 unrestricted campaign. The point, however, of constructing the microdata file is to collapse the daily data into 28 time series points, one for each month of the war. The justification for collapsing the data into monthly averages is it reduces the significant day-to-day variation. It also makes the time dimension, i.e., monthly, the same as was used for planning and evaluation by the German Admiralty.

Let t index the month/year combination, i.e., t = 8/1914, 9/1914, …, 1/1917, which can be renumbered as t = 1,2, …, 28. In a given month, there are $i = 1 \ldots N_t$ total ships sunk which will sometimes be referred to as events. To calculate the tonnage lost that month:

$$Tons_t = \sum_{i=1}^{Nt} Tons_{it} \qquad t = 1,...,28 \text{ and } i = 1,...,N_t \qquad (2)$$

The remaining variables are all monthly percentages. For example, the percentage of total ships sunk that are British in each month is

$$British_t = \sum_{i=1}^{Nt} BSHIP_{it}/_{Nt} \qquad t = 1,...,28 \text{ and } i = 1,...,N_t \qquad (3)$$

where $BSHIP_{it}$ is a dummy variable indicating if the i^{th} ship sunk in the t^{th} month is British. The same procedure is used to yield monthly percentages for the location of the sinking, which weapon was used, and so on.

One consequence of collapsing the microdata is how one generates data for subsets of the entire time series data. Assume one wants to examine the data holding one of the dummy or categorical variables constant. For example, suppose one wants to ask a specific question for a specific county. Something like "What is the distribution of types of weapons used to sink British ships?" In the microdata, one simply turns on the dummy variable for "British" and then computes the distribution. In the collapsed data, there is no way to separate out the distribution of weapons used on British ships exclusively. To answer the question in the time series framework, a solution is to recreate the time series dataset using only British data, i.e., the microdata is collapsed using only the observations for which the variable *NATIONALTY* equals 1; French data would be generated if the collapsing was based on *NATIONALITY* = 2. Similarly, collapsing the microdata when *LOCATION* equals 1 allows for a time-series-based discussion of the U-boat campaign confined just to the North Sea. Fortunately, transforming the microdata into time series observations is straightforward and easily accomplished. Only some of the major subsets will be fully analyzed.

The measurement of U-boats proceeded along two lines. The first approach used data from Michelsen (1925). Andreas Michelsen was one of the Commanders of Submarine Forces towards the end of the war. He reported on U-boats in one of three states on the 10^{th} of each month: at sea; at base; or at repair. The data is further broken down by geographic region. No collapsing of his data is necessary because it is already in a monthly dimension. The data he presents is from an official source which is generally accepted for its accuracy, thus making for a good starting point.

Developing the concept of "U-boat days" gives a more complete view of the workings of the U-boat campaign. U-boat days contain information on the number of U-boats at sea as well as the length of the patrols each U-boat has experienced. This creates a distinction between extensive variation in the use of U-boats and the intensive variation in the use of U-boats. The extensive variation comes from changing the number of U-boats at sea. Intensive variation stems from changes in the duration of the patrols of U-boats at sea. Michelsen 's(1925) data cannot capture the distinction. For example, consider the measurements of U-boat activity in October, 1915. On the 10th of the month, the author reported seven U-boats to be at sea, each of which had a patrol of 1 day, the 10th. Seven total U-boats at sea accounts for the measurement of the extensive margin. The seven days' worth of total patrolling quantifies the intensive margin of variation in U-boats. Total number of U-boat days is then 7. The average number of U-boats at sea per day is also 7. Average duration of a U-boat patrol is 1 day because the author only reported the activity of the U-boats on a single day of the month. In fact, the average duration must always be 1 for all of the author's data. Essentially, no measure of the intensive margin is observed and hence its contribution to marginal productivity is unavailable. This forms the major limitation to using the data Michelsen (1925) reports. What if on some other day, say the 22nd of October, 1915, a different number of U-boats were at sea. For a more accurate measurement of U-boat activity, it matters whether the number is greater than or less than 7. Furthermore, even if exactly 7 boats were patrolling, it matters whether they were the same 7 U-boats. It also matters how long had each of the U-boats been at sea.

To illustrate the calculation of total U-boat days, suppose one has data on the 10th and 11th of October, 1915, organized in the following way: 7 U-boats, labeled U1 to U7, were patrolling on the 10th, while on the 11th, the number of boats at sea was eight. In addition to the new U-boat at sea on the 11th (labeled the U8), imagine one of the boats that had been at sea on the 10th (say, the U7) returned to base on the 11th and a different boat took its place and started patrolling on the 11th (labeled the U9). Then total U-boat days equals 15: seven days on the 10th and eight days on the 11th of October, 1915. Dividing total U-boat days by the number of days, finds an average of 7.5 U-boats per day were patrolling the waters. Dividing total U-boat days by the number of (unique) U-boats (15/9) returns an average duration of a U-boat patrol, equal to 1.67. The average number of U-boats per day that are at sea is the extensive margin. Altering this number by either increasing or decreasing the number U-boats at sea is extensive variation in the use of U-boats. The average duration of patrols for U-boats at sea is the intensive margin. Altering this number by either increasing or decreasing the length of the U-boats patrols is intensive variation in the use of U-boats.

Tallying U-boat days requires construction of a highly detailed dataset. Koerver (2009) provides a lengthy list of U-boats and the start and end dates of their various patrols over the course of the war. The data Koerver presents comes from the Holy Grail of World War One's data on U-boats: Spindler (1932, 1933, 1934, 1941, and 1966). For each individual U-boat in a given month, the start and end dates of its patrols are recorded. Counting the individual U-boats that were at sea at some point in the given month gives the total number of unique U-boats. Summing the lengths of an individual U-boat's patrols gives the total number of days that the U-boat was at sea. It can vary between 1 and 31. Summing up the total number of days at sea for each unique U-boat produces total U-boat days. For example, if in a month with 30 days and 5 U-boats, the maximum number of U-boat days is 150; the minimum is 5. For arguments sake, assume 3 of the U-boats spent 10 days at sea and 2 of the U-boats spent 15 days at sea so total U-boat days equals 60. Dividing by number of days in the month, one gets the average number of U-boats at sea per day. In the example, that equals 2. On any given day in the month, 2 U-boats were at sea. This is, again, the extensive measure of U-boat activity because it speaks to the number of boats in the sea, able to sink ships. The average duration of a patrol during the month emerges when total U-boat days are divided by the number of U-boats, in this case equaling 12. On average, during this month, U-boats spent 12 days at sea. This quantifies the intensive measure of U-boat activity because it speaks to the degree of capacity of the U-boat is being used. Assuming all 5 U-boats could spend all 30 days at sea, then their degree of capacity is only 40%.

MEASURING THE EFFECTIVENESS OF GERMANY U-BOATS DURING THE GREAT WAR

U-boats, the Prize Rules, and Unrestricted U-boat Warfare

The question facing Germany was how were the U-boats to be used? To exploit the inherent stealth of the weapon or adhere to the rules of naval engagement? For many years, the relationship between warships and merchant ships were regulated by the so-called "prize rules" or "cruiser rules." When a warship, typically a cruiser, encountered an enemy merchant ship, a shot across the ship's bow signaled it should heave-ho. The merchant ship was then boarded and searched. If the ship was from a neutral country, it was released. If it was a combatant's ship carrying no contraband, it was also released. If it carried contraband, possession of the enemy ship was taken. The responsibility of ensuring the safety of the crew and any passengers migrated to the warship. The enemy merchant ship was not to be sunk, rather it was escorted to a port as a "prize."

The fighting of a war against merchant shipping by the prize rules was practical when the warship was a surface-based warship. This changed with the development of a new kind of warship, the U-boat. Surfacing to fire the initial shot immediately put the U-boat in a vulnerable position as it had no armoring. The next step of boarding and searching was only slightly more difficult for a U-boat, but it was still surfaced and essentially defenseless. Ensuring the safety of the crew and taking the ship to port was problematic for the U-boat. The U-boat could not take the crew aboard so, at best, the crew could take to the lifeboats. The ship would have to be sunk as towing it to a port was impossible.

The issue of the unrestricted submarine warfare was then a question of adherence to the prize rules. If the U-boats were to sink ships without warning, loss of life was nearly guaranteed. Likewise, the ability of the U-boat to discern the ship's neutrality was less than perfect. Being submerged with only a periscope for information gathering, it was conceivable a neutral ship might be sunk by mistake. The situation was complicated by the British government's decision to fly false flags. Thus, the inadvertent sinking of a neutral merchant ship was also nearly certain to occur on a regular basis. Taken together, the loss of life and sinking of neutrals caused the outcry that Germany's policy of giving no warning prior to sinking signaled their moral indecency.

Discussion of Trends

The data portrayed in Figure 1 show the trends in ships sunk and tonnage lost from January, 1915, through January, 1917. The figures aggregate losses of both warships and merchant ships for all allied combatants and all neutrals. The justification for beginning the time series in January, 1915, rather than September, 1914 stems from the learning occurring in the German Admiralty. The viewpoint of U-boats as simply being a strictly defensive weapon to one with a decidedly offensive capability evolved. For example, it was only discovered in September, 1914, that the ocean-going U-boats could circle around England successfully.

The first conclusion to take away from Figure 1 is the tonnage lost and number ships sunk grows noticeably over the 2-year period. In January, 1915, the tonnage lost is a little over 32,000 GRT and the number of ships is 8. By January, 1917, lost tonnage grew to 375,000 GRT and number of ships rose to 206. Consultation of Table 2 indicates over the 24 months, the mean tonnage lost was 158,000 GRT with an average of 88 ships lost. The overall time trend is quantified in Table 3. From January, 1915, to January, 1917, the monthly increase in tonnage sunk rose by 12,382 GRT while for ships sunk the increase was 8. Note the $(R\ bar)^2$ for both time trends for the entire 2 years is not particularly satisfying for either tons lost or ships lost.

An overall trend is not, however, the most important story of Figure 1. Instead, the data suggest that three distinct stages existed over that period. The first stage starts with the beginning of the data and lasts until September, 1915, the period covering Germany's first campaign of unrestricted U-boat warfare. A more accurate description of the campaign would have been a "no warning given to enemy merchant ships before being attacked" campaign. Anticipated by January, 1915, the codification of the U-boats' rules of engagement were officially announced in February. The policy defined the waters around Great Britain, Ireland and the whole of the English Channel as a War Zone. The area is quantified in Figure 2. In the War Zone, all enemy merchant vessels would be "destroyed" with no guarantee of the crew's safety. Ships flying neutral colors were immune from attack. The same restriction applied to both hospital ships (unless being used for troop transport) and ships connected to the Belgian Relief Commission. Neutral countries with merchant shipping were warned that by entering the War Zone, they were running the risk of an accidental sinking. The U-boat commanders were encouraged to prosecute the campaign "with all possible vigor." The High Command also indicated to the commanders if they were exercising great care, they would not be held responsible for mistakes in judgement, e.g., inadvertently sinking a neutral merchant ship.

The time trend regression in Table 3 covering the 9-month period of unrestricted activity paints a substantially different picture when compared to the entire period. The monthly increase during the campaign was 20,518 GRT. The monthly increase in sunk ships reached 13 during the unrestricted campaign. Please note the large improvement in $(R\ bar)^2$ compared with the overall regression results. The infamous *Lusitania* sinking occurred during the campaign, in May, 1915. The ship was torpedoed without warning resulting in the loss of 1,198 lives, including 124 Americans. While this incident was widely condemned, particularly by U.S. President Woodrow Wilson, the unrestricted campaign was not immediately suspended. There were growing concerns by German politicians, particularly German Chancellor Theobald von Bethmann-Hollweg that continuing the unrestricted campaign would ensure America's entry into the conflict. Ultimately, the campaign was suspended in September, 1915.

The second stage, extending from September, 1915 to May, 1916 can only be characterized as indecisive. The time trend for both tonnages lost and ships sunk is highly insignificant with poor measures of total explained variation in the data. This mostly reflects the divergent views held by the political and military leadership in Germany. Contained within the period of September, 1915 and June, 1916, is a generally unknown, second unrestricted offensive against allied merchant shipping. However, the rules of engagement changed to include additional exclusions. Enemy merchant shipping in the War Zone could still be attacked without warning but if the ships were outside of the War Zone, attack without warning was only permissible if the ship was armed. Enemy passenger steamers could not be attacked under any circumstances. Figure 1 provides vivid evidence of the effect on merchant shipping owing to the short resumption of a no warning campaign. Towards the end of April, the new Commander-in-Chief of the High Seas Fleet, Reinhard Scheer, recalled all ocean-going and costal attack U-boats back to base. The campaign against merchant shipping was effectively ended; the U-boats returned to the role of attacking enemy warships exclusively. Again, Figure 1 shows the effect on shipping losses of ending the second unrestricted campaign.

The third stage, starting in June, 1916 and continuing until the end of the data in January, 1917 shows a resumption of an aggressive use of U-boats towards merchant shipping. After the Battle of Jutland in May, 1916, Admiral Scheer concluded the British blockade of Germany could not be broken and the only choice left was to resume attacking merchant shipping. However, the policy of unrestricted attacks did not commence due to opposition from Foreign Minister Gottlieb von Jagow. His objections followed those of the past: concern over drawing the U.S. into the conflict. Finally, it was decided that a campaign following the Prize Rules would be started. Scheer was convinced an unrestricted policy would soon be in place and thereby agreed with a wink. That is from June, 1916 on, the use of U-boats was ramping up to the presumed start of more bellicose rules of engagement. The time trend in Table 3 confirms the aggressive increase in the sinking of ships and tonnage. The monthly increase in tonnage sunk was twice the rate during the first campaign at 41,000 GRT while losses of ships is up to 20. Again, note the relatively high $(R\ bar)^2$ for the June, 1916 to January, 1917 period.

REGRESSION MODELING OF PRODUCTION

With the time series data constructed and some rudimentary analysis of the data conducted, the estimation of the production function developed earlier can be taken up. The dependent variable can be either $Tons_t$ or $Ships_t$. In the following discussion, equations will be expressed for $Tons_t$ only, but when results are introduced, regressions for both variables will be presented.

The two major inputs are the daily average number of U-boats at sea, $AVGUBSDAY_t$,and the average number of days U-boats are at sea, $AVGDAYSUB_t$. The variable $AVGUBSDAY_t$ measures of the extensive use of U-boats while $AVGDAYSUB_t$ measures the intensive use of U-boats. Beyond concentrating one's attention on these two variables, one needs to account for the effects of the other variables specified earlier. The type of U-boat, the type of weapon used and whether warning was given are the other variables included in the general production function suggested in Section II. These variables will generate shift parameters.

For type of U-boat, the 2 new coastal types introduced during 1915 will be included in the regression equations. Type B will denote the coastal attack sub and Type C will denote the coastal minelayers. As to effect, it is unclear how the new coastal U-boats will affect sinkings. To the extent ships are clustered around ports and the shoreline, coastal U-boats would produce a positive effect on sinkings as more targets present themselves. However, a greater presence of ships also means a greater possibility of the U-boat sinking by either ramming or striking an Allied mine. If this effect dominates, then adding costal U-boats might reduce the sinking of ships.

The next issue to address is capturing the unique characteristic of U-boats as a weapon's platform: their stealth. Stealth comes from the ability of the U-boat's ability to keep its presence unknown to the enemy by remaining submerged. In addition to remaining hidden to enemy ships, two of the weapons at the disposal of the U-boat can be used without disclosing the U-boat's position, augmenting its stealth. The variables that serve to capture the stealth inherent in a U-boat are: *No Warning*, *Unrestricted 1*, and *Unrestricted 2*. The first, whether warning was given, is a decision made by the captain. To the extent he chooses to remain submerged prior to launching his attack and is therefore freed of the need to display his vulnerable U-boat before the attack, he should succeed in sink more ships. The second and third variables control for the political dimension of the use of U-boat stealth. To the extent giving no warning raises the number of ships sunk, then changing the rules of engagement to allow captains to sink ships without warning should raise the number of ships descending to the deep. The same positive effect should apply to tonnage lost as well. Both effects are noted in Figure 1.

The choice of weaponry should also prove important to include in the estimation exercise. The ability to use a torpedo should cause captains to pursue ships they would otherwise disregard. Because a torpedo can be used and maintain stealth, the captain is free to expand the range of ships he can sink. Since larger merchant ships were frequently armed, they could easily sink an exposed U-boat and thereby discouraging U-boat captains from pursuing them. Additional disincentive to engaged larger merchant ships when a warning must be given was their ability to ram an espoused U-boat. The effect then should be positive and probably more important in explaining the amount of tonnage than for explaining sinkings. With a mine, the physical connection between a specific ship and the use of the weapon is less direct but it still needs to be factored into the estimation. To the extent a mine is hidden and unknown to the captain of the ship, areas where ships had operated with impunity are now areas where destruction can occur without warning. Additionally, the U-boat can deposit the mine with less notice than for a minelayer. The effect of great use of mines should have a positive effect on losses; likely more so for ships than for tonnage.

A Cobb-Douglas form is chosen to express the production function and is given below:

$$TONS_t = A(AVGUBSDAY_t)^{\alpha}(AVGDAYSUB_t)^{\beta} \text{ given the shift parameters} \quad t = 1,\ldots 28 \tag{4}$$

Using the standard technique for estimating a Cobb-Douglas production function, take the log of the above equation and add a vector for the shift parameters and a random error term, producing a log linear equation.

$$\ln TONS_t = \ln A + \alpha \ln(AVGUBSDAY_t) + \beta \ln(AVGDAYSUB_t) + \gamma X_t + \varepsilon_t \qquad t = 1,\dots 28 \qquad (5)$$

The above equation will be estimated by ordinary least squares. Since the framework of the analysis is a time series, one should expect some degree of autocorrelation. The Durbin-Watson statistic will be inspected. If autocorrelation is in fact an issue, then the Cochrane–Orcutt procedure will be employed (Greene, 2011; Cochrane & Orcutt, 1949).

RESULTS & DISCUSSION

Table 4 contains the parameter estimates of different specification of the log-linear production function, where the model is estimated using time series techniques. The two most important variables are the average number of U-boats per day and the average number of days each U-boat is at sea. The sign of the first should be positive and possibly strongly positive. More U-boats at sea leads to a higher probability of an encounter with an enemy ship, raising the probability of a sinking. A possible argument could be made that more U-boats at sea might have a negative effect on losses by deterring shipping in general. That is, neutral merchant shipping, under the threat of sinking by the U-boats, might choose to not expose themselves to the threat by staying in port. To be sure, the German Admiralty had pinned some hope on this effect. But as for a lasting deterrent, the economic necessity of being at sea for merchant shipping suggests the negative influence would be small or zero. Signing the average duration of patrols proves more difficult. For a given number of U-boats at sea, the amount of U-boat days would rise if each U-boat were to remain at sea longer, thereby raising the probability of an encounter and thus sinkings. The coefficient would therefore show a positive effect. On the other hand, extending the length of patrols increases the fatigue of the crew, making them less efficient, leading to a negative effect on sinkings.

The first two columns show the estimates for the simplest model; one that includes only $AVGUBSDAY_t$ and $AVGDAYSUB_t$. When the dependent variable is tons, the (R bar)^2 is a modest but acceptable 0.66 . Most importantly, the average number of U-boats in the water per day is a positive and very significant determinant of the number of ships sunk and the amount of shipping tonnage lost. The parameter estimate is both positive and highly significant. Recalling the parameters from the Cobb-Douglas are the output elasticities, the interpretation would be a 1% increase in the average number of U-boats patrolling per day would cause an increase in tonnage sunk of 1.3%. The impact of the duration of patrols is highly insignificant.

The OLS procedure for the *Ships* equation suffered noticeable autocorrelation as indicated by both the Durbin-Watson test and the Breusch-Godfrey test, hence rendering the OLS procedure inefficient. This necessitated the re-estimation of the equation, with an appropriate corrective procedure. The Cochrane-Orcutt transformation, which is a generalized least squares technique, was employed (Greene, 2011; Cochrane & Orcutt, 1949) . Although not reported in Table 4, the OLS estimates where larger with a greater degree of significance and a very much larger (R bar)^2. The application of the Cochrane-Orcutt technique is therefore important in obtaining robust estimates. That said, the results are quite similar to those in the *Tons* equation. The average number of U-boats plays a highly significant role in the number of ships sunk whereas the average duration of patrols has no effect. The (R bar)^2 is much smaller than for the tons equation.

The first, most important control variables to introduce into the regression equation are the ones signaling a change in policy by the German government. As noted above, two unrestricted, i.e., no warning, crusades occurred. One from February, 1915 to September, 1915 and a second from February, 1916 to April, 1916. Dummy variables were constructed to cover these 2 periods, called Unrestricted 1 and Unrestricted 2, respectively, and inserted in the equation. Since the effect of these policies are to open the official door to sinking merchant ships without warning, the control variable for the percentage of ships sunk without warning, name No Warning, is included as well. As noted above, the likelihood of pursuing ships that previously were passed up owing to the need to provide warning should serve to increase the number of ships sunk. Hence the hypothesized effect of all three variables should be positive. As to whether a no warning policy causes larger ships on average to be sunk, thereby effecting relatively larger tonnage losses, intuition provides little insight.

When these variables are added to the estimation, mixed results appear. In the *Tons* equation the effect for average U-boats per day is positive, significant and much larger, rising to 2.12. This suggests that after incorporating controls for German policy decisions, the impact of having many U-boats at sea is strengthened. Now a 10% increase in the average number of U-boats produces a 21% rise in tonnage lost as opposed to the 12% increase in the initial equation. Unhappily, the parameter estimate for average duration continues in its insignificance. As to the newly introduced variables, the higher the percentage of monthly sinkings conducted without warning, the higher the amount of tonnage lost. Apparently, as U-boat captains exercise the policy of no warning, they do in fact, sink relatively larger ships. It seems that maintaining stealth allows for larger ships to be sunk. The effect is also strongly significant. Surprisingly, nether dummy variable designed to capture the changes in official policy demonstrate any degree of significance. Experimenting with subsets of the three variables shows that the only significance coefficient is the one associated with the no warning variable. Please note the rather large improvement in explanatory power of the model as (R bar)^2 improves perceptibly over the simple specification.

Quite a different story emerges for the *Ships* equation. The Durbin-Watson statistic and the Breusch-Godfrey test require the Cochrane-Orcutt transformation. The first, and most distressing, observation is that both direct measures of U-boat productivity become insignificant. The two dummy variables for the two no warning campaigns are positive and significant. This is consistent with the nature of the campaign. The U-boats were given the freedom to sink ships without warning, independent of tonnage. The change in policy stimulated a more aggressive approach to ships; it did not necessarily stimulate a greater effort to sink larger ships, although the results for the *Tons* equation suggest that this did happen. The (R bar)^2 deteriorates to 19 percent.

The next set of variables to test in isolation, i.e., without the previous 3 variables included, is the type of U-boat being used. The questions are “Did using relatively more coastal U-boats help the goal of sink ships? And were attack U-boats more or less helpful compared to minelaying U-boats?” In the tonnage equation, the same story repeats itself for the extensive vs intensive distinction. More U-boats result in a statistically significant and positive effect on tonnage losses but longer patrols do not have a significant impact. Both types of coastal U-boats fail to register a significant effect at either the 1% or 5% level of significance. Dropping down to a 10% significance level leads to a ruling of a positive and significant coefficient for Type C U-boats. Furthermore, if the equation is re-estimated using only Type C U-boats, the coefficient is 1.298 and is significant at the 5% level. It appears that weak evidence exists for adding a new type of weapon, a U-boat that could lay mines, enhances the amount of tonnage lost.

For the ships sunk equation, the coefficient for number of U-boats is positive and significant while the duration coefficient proves still unimportant. Both types of coastal U-boats are shown to be insignificant with respect to sinking ships. If one again considers raising the level of significance to 10%, then both coefficients would be significant. Unfortunately, they would be significant and negative. The unsurprising interpretation would be that ocean-going U-boats are better at sinking ships compared to coastal types. The overall explanatory power of the equation for tons is laudable at 70% but the *Ships* equation continues to possess low explanatory power.

The last set of variables to consider involves the weaponry employed. As noted earlier, while U-boats can utilize gunfire or scuttling to sink a ship, the unique weaponry associated with a U-boat are torpedoes and mines. The goal of including these variables is to measure if a higher relative usage of torpedoes results in greater tonnage and/or ships sunk. It seems reasonable to presume greater use of the weapon that can sink a ship unannounced due to the existence of U-boats would produce greater losses, particularly during times when the U-boat is not obligated to supply a warning. To a lesser degree, the same reasoning can be applied to mines. Mines can and have been laid by ships but, as argued above, a U-boat can lay mines without attracting as much attention to the location of the mines.

For the consistently better performing equation, tonnage, number of U-boats is significant; duration of patrols is not. As to weapon used, both torpedoes and mines are positive, significant, with relatively large effects. The *Ships* equation’s poor performance continues. Neither torpedoes nor mines have a significant effect on number of ships sunk. In fact, the influence of the number of U-boats falls in significance from the 1% level to the 5% level.

The final version of the estimation exercise inserts all the variables discussed to this point into the regressions. For the *Tons* equation, number of U-boats continues its highly significant effect with a 10% increase in average number of U-boats leading to an extra 16% tonnage sent to the bottom of the sea. The duration of patrols stays insignificant. Neither type of coastal U-boat demonstrates any significance. Both types of weapons play a positive and significant role in sinking tonnage. Finally, the second, shorter unrestricted campaign is significant but of an unexpected sign.

The results for the *Ships* equation are quite perplexing. First, the tests for autocorrelation had up to this point shown a need to correct for first degree autocorrelation, which was done by the Cochrane-Orcutt transformation. In the full specification, the tests indicate autocorrelation is absent so the OLS estimates are presented. As to the explanatory variables, number of U-boats remains positive and significant while duration plays no role. Mines and the first unrestricted crusade are both significant and of the expected sign. Mines were significant and positive in the *Tons* equation, so the significance in the *Ships* equation is reassuring. The dummy variable for the first unrestricted campaign is significant for ships whereas for tons it does not contribute to larger losses. This appears quite reasonable and consistent with the results when only policy changes were considered.

CONCLUSIONS & FUTURE RESEARCH

The clear conclusion apparent from the empirical results indicates variation at the extensive margin consistently increases the amount of tonnage and ships sunk across all but one of the specifications. For the tonnage equations, the impact of a 10% increase in the average number of U-boats ranges from a low of a 13% increase in tonnage losses to a high of 21%. The final specification gives an estimate of an 19% loss. This in turn translates in a marginal product of 22,483 additional tonnage lost. For ships, the range for a 10% increase in the average number of U-boats was from 8% to 16%. The 16% value comes from the final specification so converting this into a marginal product results in an additional 10 ships lost.

Variation at the intensive margin consistently shows no impact on either the amount of tonnage lost or the number of ships sunk. This may, on reflection, be the correct result. To the extent the U-boats are homogenous within type (an assumption that was made), then their capacity to be at sea would be homogenous as well. If the decision taken was to patrol all U-boats about the same number of days, then there would be no intensive variation which then should leave no trace in the regression analysis. This point is supported by examining the coefficient of variation. It is considerably larger for the average number of U-boats per day (0.567) than for the average number of days spent patrolling (0.141).

Most of the remaining results are mixed. The type of U-boat seems completely irrelevant to the losses imposed on the enemy. That torpedoes are significant for tons but not ships appear reasonable. The value of sinking a relatively large ship might encourage the use of complete stealth via a torpedo and no warning. A relatively smaller ship might not require or encourage the use of the weapon with the most stealth. Indeed, having a limited amount of torpedoes and many gun shells and bombs, the use of a warning with either gunfire or scuttling was likely a common combination to use to sink smaller ships. This is confirmed in the microdata. If the ship is small, less than 1,000 tons, they were overwhelmingly stopped (81%) and then sunk by bomb or gunfire (60%). Mines seem to play a significant role in the sinking of tonnage and ships. Finally, the impact of the two unrestricted campaigns seems to only be important for explaining the number of ships lost but not the tonnage sunk. In general, the regressions' ability to explain variation in the tonnage lost systematically and significantly exceeds the explanation of variation of the number of ships lost.

Based on these results, Germany's strategy for disrupting enemy and neutral merchant shipping should include having on average more U-boats at sea. Those additional U-boats as well as the ones already at sea should make greater use of torpedoes and mines in attacking shipping. Importantly, it seems to not matter if warning is given to the ship; no significant improvement in tonnage lost or ships sunk results from not giving a warning. The greater amount of sinkings when the two no warning unrestricted campaigns were in place could stem more from the expansion of the type of ship the German Admiralty allowed captains to consider for sinking. Germany seems it could have a more aggressive and successful campaign from its U-boats without needing to announce a policy of "unrestricted" sinkings. Judgements about these questions are clouded by weight of the *Lusitania*; it was an extreme exception and hardly the rule. It continues to illustrate how disastrous that one decision was to the German war effort.

With respect to future research, there are many avenues to travel down. Nearly unnecessary to say, completing the assembling of the microdata set through November, 1918 is the next step. Undoubtedly, many insights will be afforded from a complete dataset. This will nearly double the number of monthly observations. In an effort to further increase the number of observations by shrinking the size of the periods by collapsing the microdata by fortnights rather than months. Completing the data to the end of the war and collapsing by fortnight would drive the number of observations to 100. Experimentation with collapsing the microdata by days may be undertaken. Next, the fact that sinking a ship is first an exercise in probability must be included. One approach would be to collapse the microdata by day and then go back and insert rows with zero on the days nothing was sunk and then use a Tobit estimation model. The "supply", the traffic on the shipping lanes, must also be incorporated. Much work remains to understand the exact interaction between the hunted and the hunters. There is a market relationship in there somewhere.

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TABLE 1
SUMMARY STATISTICS FOR MICRODATA

Variable	Mean	Standard Deviation	Minimum	Maximum
Tons	1800.61	2485.90	5.0	48,158.0
Nationality	Britain (0.498)	Norway (0.128)	France (0.115)	Italy (0.892)
Location of Sinking	North Sea (0.313)	Atlantic (0.276)	Mediterranean (0.234)	Channel (0.139)
Type of Warship	Trawler (0.440)	Minesweeper/ Minelayer (0.147)	Troopship (0.576)	Destroyer (0.524)
Type of Merchant Ship	Steamer (0.565)	Sailing Vessel (0.221)	Trawler (0.175)	Passenger Steamer (0.022)
Cargo	Coal (0.249)	Timber (0.152)	Miscellaneous (0.142)	Food Stuffs (0.136)
Casualties	16.78	74.78	0.0	1198.0
Merchant Ship Armed	0.048	0.213	0.0	1.0
No Warning	0.213	0.449	0.0	1.0
Weapon	Bomb (0.274)	Gunfire (0.239)	Torpedo (0.230)	Mine (0.143)

TABLE 2
SUMMARY STATISTICS FOR MONTHLY TIME SERIES DATA

Variable	Mean	Standard Deviation	Min	Max
Tons	158120.4	100052.9	22784.0	375319.0
Ships	88.76	61.21	8.0	206.0
AVG UBS/DAY	13.41	7.61	4.39	28.39
AVG DAYS/UB	10.29	1.45	7.46	13.28
Type B	0.189	0.156	0	0.516
Type C	0.197	0.178	0	0.583
Torpedo	0.275	0.174	0.074	0.778
Mine	0.171	0.172	0	0.583
No Warning	0.366	0.201	0.129	0.778

TABLE 3
TIME TREND REGRESSION ESTIMATES

Variable	Tons				Ships			
Period	1/15-1/17	1/15-9/15	10/15-5/16	6/16-1/17	1/15-1/17	1/15-9/15	10/15-5/16	6/16-1/17
Time	12382.5*	20518.4*	8092.0	43153.3*	7.95*	12.73**	5.33	19.71*
s.e.	3880.53	1330.4	7155.18	5895.3	3.003	3.463	2.850	3.323
$(\bar{R})^2$	0.272	0.929	0.176	0.883	0.182	0.610	0.263	0.830

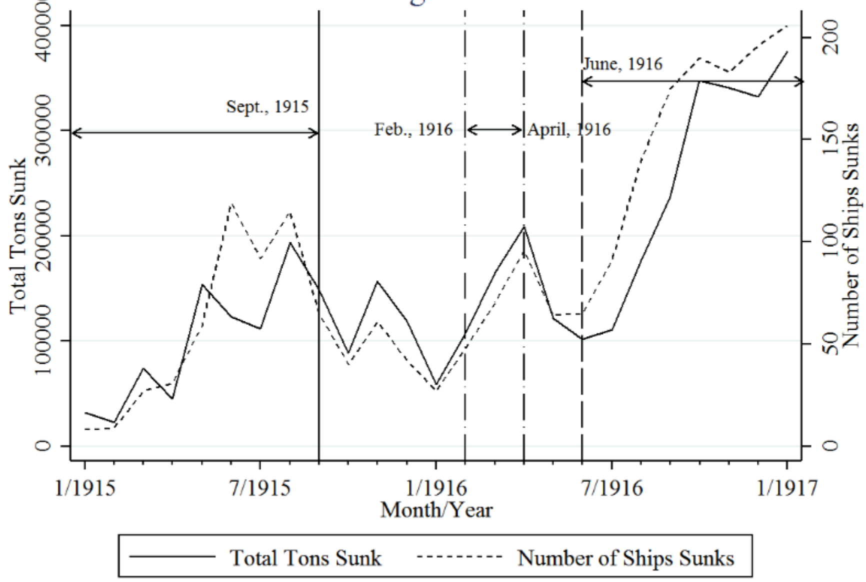
TABLE 4
PRODUCTION FUNCTION ESTIMATES[1]

	ln Tons	ln Ships[2]	ln Tons	ln Ships[2]	ln Tons	ln Ships[2]	ln Tons	ln Ships[2]	ln Tons	in Ships
ln Avg UBs/ Day	**1.298***	**1.030***	**2.117***	**0.709**	**1.396***	**0.847***	**1.881***	**0.854****	**1.906***	**1.578***
s.e.	0.222	0.231	0.338	0.489	0.309	0.233	0.255	0.416	0.435	0.352
Ln Avg Days/ UB	**-0.339**	**-0.442**	**-1.269**	**-0.376**	**-0.460**	**-0.414**	**-1.144**	**-0.386**	**-1.063**	**-0.902**
s.e.	1.042	0.558	0.982	0.848	1.008	0.517	0.869	0.734	0.0.894	0.724
No Warning			**2.533***	**-0.301**					**-2.795**	**-2.863**
s.e.			0.982	1.099					1.839	1.488
Unres 1			**0.391**	**0.348***					**0.278**	**0.532***
s.e.			0.256	0.121					0.237	0.191
Unres 2			**-0.183**	**0.549****					**-0.67***	**-1.772**
s.e.			0.385	0.243					0.375	1.302
Type B					**-0.874**	**-0.856**			**-0.242**	**0.112**
s.e.					1.094	0.440			0.964	0.780
TYPE C					**1.240**	**-0.916**			**-0.025**	**0.667**
s.e.					0.637	0.528			1.611	1.304
Torpedo							**2.208***	**1.042**	**5.335***	**2.707**
s.e.							0.683	0.818	1.749	1.416
Mine							**2.609***	**-0.185**	**6.674***	**4.185****
s.e.							0.685	0.968	2.299	1.861
No Obs.	28	26	28	26	28	26	28	26	28	28
(R bar)^2	0.663	0.277	0.741	0.196	0.697	0.174	0.784	0.234	0.820	0.913
D/W	1.127	0.876	1.614	0.917	1.356	0.963	1.232	1.066	1.811	1.759
Chi^2	1.429	5.474	0.238	6.385	0.0	4.852	2.566	4.186	0.130	0.046

1“*” is significant at the 1% level; “**” is significant at the 5% level.

2 Equations estimated using the Cochrane-Orcutt transformation to correct for first order autocorrelation.

Figure 1



Total Tons Sunk
0
100000
200000
300000
400000
Number of Ships Sunks
0
50
100
150
200
Sept., 1915
Feb., 1916
April, 1916
June, 1916
1/1915
7/1915
1/1916
7/1916
1/1917
Month/Year
Total Tons Sunk
Number of Ships Sunks


SCALE OF MILES
0 50 100 200 300
SHADED PORTION INDICATES THE WAR ZONE PROCLAIMED BY GERMANY FEB. 18. 1915
ATLANTIC OCEAN
SHETLAND IS.
ORKNEY IS.
HEBRIDES
BERGEN
STAVANGER
NORWAY
SCOTLAND
ABERDEEN
EDINBURG
NORTH SEA
IRELAND
BELFAST
IRISH SEA
DUBLIN
SUNDERLAND
QUEENSTOWN
LIVERPOOL
ENGLAND
HULL
WALES
LUSITANIA SUNK MAY 7. 1915
FALABA SUNK MARCH 28. 1915
SCILLY IS
GULFLIGHT TORPEDOED MAY 1. 1915
HELIGOLAND
CARDIFF
YARMOUTH
HOLLAND
HAMBURG
LONDON
AMSTERDAM
DOVER
PLYMOUTH
PORTSMOUTH
CALAIS
ANTWERP
BELGIUM
ENGLISH CHANNEL
CHERBOURG
BREST
HAVRE
PARIS
GERMANY
FRANCE

Figure 2